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THE THIRD LINKAGE GROUP IN OENOTHERA[1]

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In 1921 there appeared in my cultures of *Oenothera Lamarckiana* a new flower color, which I have designated "old-gold" (*vetaurea*). The origin of this beautiful mutation was described in the *Journal of Heredity* in October, 1921, and it was there suggested that the factor for old-gold might be in linkage-group I, since the original family in which it occurred showed in its *Lamarckiana*-like component a ratio of 40 yellow : 13 old-gold, while in the *decipiens*-like component, the few plants that bloomed consisted of 2 yellow and 7 old-gold. The grouping into *Lamarckiana*-like and *decipiens*-like was understood to rest on the presence and absence, respectively, of a zygote lethal located in linkage-group I.

It was recognized that the number of *decipiens* plants which flowered was too small to allow any dependence to be placed in the suggested linkage of old-gold with a first-chromosome lethal, and consequently, no surprise was caused by the demonstration in the next year's cultures, and in very extensive experiments since, that the factor for old-gold is independent of the large group of known factors which make up the first linkage group.

Old-gold has also been found to be independent of the second-chromosome factor pair, long style (B_r) *versus brevistylis* (b_r). Consequently, the factor for old-gold is the first discovered factor of a third linkage group in Oenothera. The numerous experiments which demonstrate this independence of old-gold from both of the previously known groups, I and II, will be reported in full in *Genetics* for May, 1926.

The main purpose of the present paper is not, however, to give an advance report on the status of old-gold as a third-chromosome factor, but to give a brief preliminary account of the finding of a second factor in this third linkage group.

Late in the summer of 1923 a plant was found in pedigree No. 22159 (the F_2 from a cross between a yellow-flowered mut. *pervirens* and an old-gold-flowered *Lamarckiana*), which had the old-gold flower color

seemingly associated with the green stems and green buds of mut. *pervirens*. As this was a desired combination and it was too late in the season to make hand-pollinations, unguarded seeds were collected and sown in 1924 under the pedigree No. 23137. The offspring showed a split into *Lamarckiana*, *erythrina* and *decipiens*, presumably possessing, respectively, two, one and no lethal factors, and there was also a segregation to *pervirens*, thus showing that the parent was probably not a *pervirens*, as supposed, but a heterozygote which failed to show red pigmentation of stems and buds owing to the lateness of its development.

How much of the unique variation in this family was due to outcrossing could not be determined, of course, but since the mother had the recessive old-gold flower color, all plants which had the dominant yellow flowers could be identified as certainly due to outcrossing, and it may be assumed, fairly, I think, that the outcrosses to old-gold were probably considerably less numerous than the outcrosses to yellow, since there were many more yellow-flowered plants than old-gold ones available for such outcrossing. As there were 62 yellow-flowered plants and 62 old-gold ones in this family, it may be concluded that cross-fertilization was the strongly predominant method of breeding in the long-styled parent of this progeny.

On August 22, 1924, one of the old-gold *Lamarckianas* in this family was seen to have one to several supernumerary petals in many of its flowers, associated with greater or less modification of the pistils, while in most of the other flowers on this plant, which had no supernumerary petals, the pistil was nevertheless modified by having the stylar "tube" slightly split, or showing partial separation of one or more of the stigmatic lobes. An examination of the other plants in this family showed that no fewer than seven individuals in the family had supernumerary petals and modified stigmas, and an eighth had modified stigmas, but among the few flowers which were observed on this last plant, no extra petals were seen. As all of these eight plants had old-gold flowers, it is supposed that they resulted from self-fertilization, even though the mother had been unguarded and had been therefore largely cross-fertilized.

So far as I am aware, this is the first case of "double" flowers which has been reported in the Oenotheras. The new form which thus made its sudden appearance has been designated mut. *supplena*. Of these first eight specimens of mut. *supplena* two were *Lamarckiana*-like in form, two were *erythrina* in form, and five had the form of *decipiens*. As these three vegetative forms are generally known to differ in the number of zygote lethals present, and these lethals belong to the first linkage group, it was at once inferred that the *supplena* factor was independent of the first linkage group. In favor of the same interpretation was the fact that the frequency of occurrence of the new type must have agreed approximately with that of a typical Mendelian recessive, which would not have

been the case, if it were linked with the first-chromosome lethals. The question at once presented itself as to whether this newly discovered *supplena* factor was linked with *brevistylis* in group II, with *vetaurea* in group III, or whether it represented the first factor of a fourth linkage group.

A fortunate circumstance has made it possible to determine the answer to this question a year earlier than had been anticipated. One of the yellow-flowered sibs of these eight plants of mut. *supplena* had resulted from the chance meeting of a sperm carrying the factors for *rubricalyx* buds, yellow flower color and single flowers, with an egg having the factors for green-hypanthium buds, old-gold flower color and double flowers (mut. *supplena*). The offspring produced by self-fertilization of this plant have been grown the past season (1925), under the pedigree number 24130, and have consisted of 94 plants with *single yellow* flowers, and 22 with *double old-gold* flowers, thus demonstrating the close linkage of the factors for old-gold and doubling. All plants of both groups had *rubricalyx* buds, though the parent was just as surely heterozygous for bud color as for flower color and for doubleness. The failure to segregate with respect to bud color showed that the plants, all of which had the habit of vigorous *Lamarckiana*, probably possessed both of the zygote lethals l_1 and l_2, or at least, that the egg which supplied the factors for old-gold and double flowers, also carried one of the *Lamarckiana* zygote lethals linked with its factor for green-hypanthium buds; otherwise, green-hypanthium buds would likewise have segregated out in this family. The fact that the old-gold and doubling segregated from their dominant allelomorphs, yellow and singleness, in approximately a monohybrid ratio, while the bud color did not segregate, confirms the conclusion that the old-gold and doubleness are inherited independently of linkage-group I and its contained lethal factors.

The discovery of this third linkage group in Oenothera promises to be of great value in helping to rationalize our interpretations of the genetical phenomena in this genus. The extensive breedings with old-gold indicate that there is no such general presence of lethal factors in linkage-group III, as there is in group I, and consequently, the work with group III fits the Mendelian model much more closely than does that with group I. Genetical work with three linkage groups may prove a most important factor in determining the validity of Cleland's[2] suggestion that chromosome cohesions may constitute the cytological basis of linkage and crossing over in this group of plants, which was offered as an alternative to the hypothesis I have favored,[3] namely, that linkage in the Oenotheras results, as in other organisms, from the inclusion of the linked genes within a single chromosome pair. Only in *Oenothera biennis* has Cleland found, thus far, more than one ring of temporarily united chro-

mosomes. So long as all the known factors except *brevistylis* were included in a single linkage group, one ring of united chromosomes might seem to provide an adequate mechanism for the observed linkage, except that about twice as many factors have been found in the first linkage group as there are haploid chromosomes in Oenothera. Coördinated genetical and cytological studies on material characterized by three groups of linked genes, will present a number of critical tests which should make it possible to decide promptly which of the two rival hypotheses is correct.

[1] The experiments on which this paper is based, have been supported in part by grants from the American Association for the Advancement of Science and the Elizabeth Thompson Science Fund.

[2] Cleland, R. E., 1923, "Chromosome Arrangements during Meiosis in Certain Oenotheras," *Amer. Nat.*, **57**, 562–566.

[3] Shull, G. H., 1921*a*, "Mendelian or Non-Mendelian?" *Science N. S.*, **54**, 213–216; 1921*b*, "Three New Mutations in *Oenothera Lamarckiana*," *J. Heredity*, **12**, 354–363; 1923*a*, Linkage with Lethal Factors the Solution of the Oenothera Problem, *Proc. Second Internat. Cong. Eugenics*, **1**, 86–99; 1923*b*, "Further Evidence of Linkage with Crossing Over in Oenothera," *Genetics*, **8**, 154–167.

FERTILITY OF THE RAT, MUS NORVEGICUS

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A study of the fertility of the Norway rat reveals that phenomenon to be the manifestation of various other phenomena, and not a unit in itself. The fertility, defined as fruitfulness of a mating between a given male and a given female in terms of living offspring, proved in my experiments to be essentially an expression of the reproductive vigor of the individuals concerned. Further, fertility in most cases was dependent upon the vigor of the female only, for it was through her that practically all of the detracting factors came. The male rat, unless he was sterile, inseminated with a quantity and quality of secretions which were capable of producing a full-sized litter.

During the course of the experiments with which this paper deals, the reproductive vigor and growth of 459 female and 377 male rats have been studied. Their fertility has been investigated by 1445 different matings which resulted in the birth of a total of 1740 litters consisting of 10,104 young rats. These rats composed three litters of the red-eyed yellow mutant variety and their descendants through four generations of brother-sister matings. In addition, a control stock of this variety was maintained

in parallel generations by matings between animals which were second cousins and in some cases more distantly related. Also, a small stock of the fifty-second generation of Doctor Helen Dean King's closely inbred albinos was raised as a further control on certain points. The fertility of all of the rats with the exception of the controls was tested by the diallel mating of each male with each of his sisters. The control stock was tested by mating the males with females which were as distantly related to them as the stock would permit. The size of the litters permitted the testing of all animals while they were less than a year old and therefore in fair condition. My data throw no light upon senescence. Food, management and other environmental conditions were maintained as uniformly as possible. They may be safely excluded as factors in this work.

Of the factors which influence reproductive vigor, promptness of mating is important. My rats were first mated at the age of 90 days and females were remated within 48 hours following each parturition thereafter. The data show that pregnancy succeeded very promptly, an average of 3.2 days after mating. Only 22.4 per cent of the first matings required more than the average oestrous cycle of 4.6 days. In matings later than the first, about 8 days, or more than three days in excess of the average oestrous cycle, were required before pregnancy occurred. The number of delayed pregnancies fluctuated between 55.9 and 71.1 per cent in the matings at advanced ages. The tardiness of conception may have been due to the retarding influence of a short lactation period upon the return of the oestrum, coupled with the effects of the preceding pregnancy. The average time required by a female after being mated until parturition, correlated with her body weight at 90 days, gave a coefficient of -0.193 ± 0.033. This slight correlation between promptness of conceiving and general vigor was borne out by the restricted agreements between these characters in various groups of the population.

The most important measure of reproductive vigor was litter size. It appeared to be subject to extreme fluctuations caused by widely different agents. The position of a litter in the female's reproductive history was not a factor in itself in determining the number of live individuals in that litter. The age of the female at the time the litter was born was a factor, however. The average litter size increased with age to the group, 150–179 days. With advanced age the average litter became smaller, and reached its lowest level in the most advanced age groups. From a study of the factors influencing litter size, the effects of age appeared to be due principally to the fluctuations of the general vigor of the individual. The attainment of the maximum body size, before the slow accumulations of advanced age, parallelled litter size increase. Weight of female at 90 days of age and average size of litter gave a correlation coefficient of $+0.356 \pm 0.030$. Inbreeding by brother-sister matings was accompanied by a

TABLE 1

SUMMARIZED EFFECTS UPON REPRODUCTION, OF AGE OF FEMALE

	AGE OF FEMALE IN DAYS									
	90–119	120–149	150–179	180–209	210–239	240–269	270–299	300–329	330–359	ALL AGES
Number of days from mating to birth of young	25.2	29.5	30.0	29.8	28.3	29.4	30.2	30.6	30.4	29.0
Number of delayed pregnancies in per cent	22.4	57.1	66.0	61.6	59.9	55.9	68.6	66.7	71.1	56.4
Average size of litter	5.60	6.13	6.27	5.73	5.40	5.85	5.58	4.97	5.28	5.80
Percentage of stillborn rats	11.0	6.9	7.6	9.5	9.2	8.0	7.7	13.1	9.7	8.66
Percentage of litters containing stillbirths	25.1	21.7	22.5	29.1	27.7	24.5	27.1	41.3	23.0	25.57

TABLE 2

COMPARATIVE WEIGHT IN GRAMS OF THE TWO SEXES AT VARIOUS AGES, ALL GROUPS INCLUDED

	APPROXIMATE AGE IN DAYS											
	30	50	70	90	120	150	180	210	240	270	300	330
Females	38.0	76.2	106.1	129.7	174.2	183.9	201.6	197.6	200.1	202.4	197.4	229.0
Males	39.4	82.2	149.7	168.0	221.8	228.1	250.6	257.6	265.4	254.7	283.3	266.0

TABLE 3

SUMMARIZED EFFECTS OF INBREEDING ON REPRODUCTIVE VIGOR AND GROWTH IN THE VARIOUS STOCKS

GENERATION	AVERAGE MATING TIME IN DAYS	PERCENTAGE OF DELAYED MATINGS	AVERAGE LITTER SIZE	PERCENTAGE OF LITTERS CONTAINING STILLBIRTHS	WEIGHT IN GRAMS OF FEMALE AT 90 DAYS	WEIGHT IN GRAMS OF MALE AT 90 DAYS	PERCENTAGE OF FERTILE MATINGS	PERCENTAGE OF UNANALYZED STERILITY	PERCENTAGE OF STERILE MALES	PERCENTAGE OF STERILE FEMALES
First inbred	27.4	55.7	7.59	22.8	127	171	100.0	0.0	0.0	0.0
Second inbred	30.4	62.9	5.82	16.0	123	157	85.1	0.6	3.0	4.0
Third inbred	28.3	53.1	5.71	27.6	129	167	85.4	0.4	0.9	3.7
Fourth inbred	29.8	64.6	3.86	42.3	118	149	66.3	2.6	2.0	18.0
Fifty-second (King) inbred	28.2	60.0	6.08	45.8	190	228	75.0	10.0	9.1	9.1
First cross-bred	29.1	40.9	7.93	9.0			87.2	6.4	0.0	0.0
Second cross-bred	25.1	19.4	7.54	13.4			88.0	0.0	8.0	4.0
Third cross-bred	28.1	57.8	6.43	53.3	142	176	90.5	0.0	0.0	0.0
Total	29.0	56.4	5.80	25.6	129	168	83.8	1.0	2.4	5.4

steady decrease in litter size as indicated by generation averages. The seasonal variations of litter size were considerable. The months preceding June showed averages which fluctuated between 6.3 and 6.5 young per litter. June with a maximum of 7.3 was followed by a steady decline to the low point of 5.2 in October. In November and December the average litter size again increased. These variations were parallel in all years.

The size of litter in terms of live rats was decidedly affected by the presence of dead individuals. The proportion of stillborn rats and the proportion of litters containing one or more stillborn individuals varied with age in a manner which suggested an explanation for at least part of the age variation of litter size. The minimum occurred in the age group, 120–149 days. Stillbirths indicated the weakness of the female generative system at one point in the performance of its function. Inbreeding, probably through its effects upon general vigor, was accompanied by an increase in the frequency of stillbirths.

This study has indicated that in addition to the factors of reproductive vigor there existed in these rats certain sterility conditions. Of the total matings, 83.8 per cent resulted in the birth of at least one young rat. Of the remaining fruitless matings all but 1.0 per cent were explained by a permanent or temporary sterility of either mate. The causes of sterility were in a few cases malformations of the organs of generation. In most of the cases, however, it was due to pathological conditions of the reproductive system or to the indirect effect of other diseased states of which pneumonia was the most frequent. Mishaps of former pregnancies proved a common cause of sterility in females. The one per cent of unexplained sterility may have been due to incompatibility of the gametes. Its distribution in the population did not indicate, however, that it was similar in nature to the self sterility (or cross sterility) which has been described for certain plants and for the Ascidian, *Ciona intestinalis*. The method of mating employed in this work was well suited to the demonstration of the occurrence and inheritance of incompatibility of that type, had it existed.

Reproductive vigor, growth and fertility in the rat are difficult of analysis from the standpoint of their inheritance. Their variability from other causes makes the phenotype a very inadequate criterion of the individual's genetic composition. Although certain strains have segregated for marked degrees of reproductive vigor, the manner in which the segregation occurred is not clear.

A more complete account of this investigation will be published elsewhere.

INVARIANTS OF RELATIVE QUADRATIC DIFFERENTIAL FORMS

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1. It was in considering a theory of conformal Riemann geometry that my attention was directed to the theory of relative quadratic differential forms, i.e., the theory of the equations

$$\bar{G}_{\alpha\beta}\, d\bar{x}^{\alpha}\, d\bar{x}^{\beta} = (x\bar{x})^{\kappa}\, G_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta}. \tag{1.1}$$

In these equations $G_{\alpha\beta}$ and $\bar{G}_{\alpha\beta}$ depend on variables $(x^1, \ldots, x^n)$ and $(\bar{x}^1, \ldots, \bar{x}^n)$, respectively, $(x\bar{x})$ denotes the jacobian of a transformation

$$x^i = \varphi^i\,(\bar{x}^1, \ldots, \bar{x}^n) \tag{1.2}$$

and κ is an arbitrary constant called the weight of the form $G_{\alpha\beta}\, dx^{\alpha}\, dx^{\beta}$. It is assumed that $(x\bar{x})$ does not vanish identically.

Now the general invariant theory of the equations (1.1) may be handled very easily. This includes intrinsic Riemann geometry as a special case, namely when $\kappa = 0$. If, however, the condition $n\kappa + 2 = 0$ is satisfied the general theory breaks down and no simple modification seems possible. It is here that the conformal geometry arises. What is possibly needed for its development is the introduction of a device corresponding to that used in my projective theory of affinely connected manifolds.[1] But so far I have not obtained a satisfactory treatment of the conformal geometry.

2. Our theory demands that the determinant

$$\begin{vmatrix} G_{11} & \ldots & G_{1n} \\ G_{n1} & \ldots & G_{nn} \end{vmatrix} \tag{2.1}$$

does not vanish identically. Also since the functions $G_{\alpha\beta}$ occur as the coefficients of a quadratic differential form we may assume them to be symmetric, i.e., to satisfy the conditions $G_{\alpha\beta} = G_{\beta\alpha}$. Similar remarks apply to the functions $\bar{G}_{\alpha\beta}$.

Let us now write the equations (1.1) in the form

$$\bar{G}_{\alpha\beta} = (x\bar{x})^{\kappa}\, G_{\sigma\tau}\, \frac{\partial x^{\sigma}}{\partial \bar{x}^{\alpha}}\, \frac{\partial x^{\tau}}{\partial \bar{x}^{\beta}}. \tag{2.2}$$

Then our assumption regarding the determinant $|G_{\alpha\beta}|$ allows us to give these equations the contravariant form

$$\bar{G}^{\alpha\beta} = (x\bar{x})^{-\kappa}\, G^{\sigma\tau}\, \frac{\partial \bar{x}^{\alpha}}{\partial x^{\sigma}}\, \frac{\partial \bar{x}^{\beta}}{\partial x^{\tau}} \tag{2.3}$$

in which $G^{\sigma\tau}$ is the co-factor of $G_{\sigma\tau}$ in the determinant $|G_{\alpha\beta}|$ divided by $|G_{\alpha\beta}|$. From equations (2.2) we obtain

$$|\bar{G}_{\alpha\beta}| = (x\bar{x})^{n\kappa+2}\,|G_{\alpha\beta}|. \tag{2.4}$$

The jacobian $(x\bar{x})$ may now be eliminated between the equations (2.2) and (2.4) provided that the condition

$$n\kappa + 2 \neq 0 \tag{2.5}$$

is satisfied. In general this will be the case. Then

$$\bar{H}_{\alpha\beta} = H_{\sigma\tau}\frac{\partial x^{\sigma}}{\partial \bar{x}^{\alpha}}\frac{\partial x^{\tau}}{\partial \bar{x}^{\beta}} \tag{2.6}$$

where

$$H_{\sigma\tau} = \frac{G_{\sigma\tau}}{|G_{\alpha\beta}|^{r}}\,;\, r = \frac{\kappa}{n\kappa+2}. \tag{2.7}$$

The functions $\bar{H}_{\sigma\tau}$ are defined in a corresponding manner. Differentiating (2.6) we may obtain the equations for the second derivatives $\partial^2 x^i/\partial\bar{x}^{\alpha}\partial\bar{x}^{\beta}$ in the form

$$\bar{\Gamma}^{\sigma}_{\alpha\beta}\frac{\partial x^{i}}{\partial \bar{x}^{\sigma}} = \frac{\partial^2 x^{i}}{\partial\bar{x}^{\alpha}\partial\bar{x}^{\beta}} + \Gamma^{i}_{\sigma\tau}\frac{\partial x^{\sigma}}{\partial\bar{x}^{\alpha}}\frac{\partial x^{\tau}}{\partial\bar{x}^{\beta}} \tag{2.8}$$

where

$$\Gamma^{i}_{\alpha\beta} = \tfrac{1}{2}H^{i\sigma}\left(\frac{\partial H_{\alpha\sigma}}{\partial x^{\beta}} + \frac{\partial H_{\sigma\beta}}{\partial x^{\alpha}} - \frac{\partial H_{\beta\alpha}}{\partial x^{\sigma}}\right) \tag{2.9}$$

and the functions $\bar{\Gamma}^{i}_{\alpha\beta}$ have a similar definition in terms of the functions $\bar{H}_{\alpha\beta}$. An expanded form of $\Gamma^{i}_{\alpha\beta}$ is given by

$$\Gamma^{i}_{\alpha\beta} = \left\{\begin{matrix} i \\ \alpha\beta \end{matrix}\right\} - r\left(\delta^{i}_{\alpha}\left\{\begin{matrix} v \\ v\beta \end{matrix}\right\} + \delta^{i}_{\beta}\left\{\begin{matrix} v \\ v\alpha \end{matrix}\right\} - G_{\alpha\beta}G^{i\sigma}\left\{\begin{matrix} v \\ v\sigma \end{matrix}\right\}\right) \tag{2.10}$$

where

$$\left\{\begin{matrix} i \\ \alpha\beta \end{matrix}\right\} = \tfrac{1}{2}G^{i\sigma}\left(\frac{\partial G_{\alpha\sigma}}{\partial x^{\beta}} + \frac{\partial G_{\sigma\beta}}{\partial x^{\alpha}} - \frac{\partial G_{\beta\alpha}}{\partial x^{\sigma}}\right). \tag{2.11}$$

From this point on the usual methods for the formation of differential invariants may be applied. Thus a theory of extension based on the functions $\Gamma^{i}_{\alpha\beta}$ is possible. Their use, too, permits a definition of infinitesimal parallel displacement, the establishment of an invariant set of paths, the erection of systems of normal coördinates. It may be shown that the tensor $G_{\alpha\beta}$ and its extensions $G_{\alpha\beta,\gamma\delta\ldots\varepsilon}$ $(E \geq 2)$ constitute a complete set of invariants of the relative differential form $G_{\alpha\beta}dx^{\alpha}dx^{\beta}$. The first extension or covariant derivative $G_{\alpha\beta,\gamma}$ vanishes identically as in the special case of the Riemann geometry. Also the vanishing of the curvature tensor $B^{i}_{\alpha\beta\gamma}$ based on the functions $\Gamma^{i}_{\alpha\beta}$ is a necessary and sufficient

condition for the differential form $G_{\alpha\beta}dx^\alpha dx^\beta$ to be reducible to a sum of squares $a_\alpha(dx^\alpha)^2$ where $a_\alpha = +1$ or $a_\alpha = -1$ etc., etc.

3. If

$$n\kappa + 2 = 0 \tag{3.1}$$

the equations (2.4) give

$$|\bar{G}_{\alpha\beta}| = |G_{\alpha\beta}|. \tag{3.2}$$

The determinant $|G_{\alpha\beta}|$ may be variable or constant. In the case of the conformal geometry, however, the equation

$$|G_{\alpha\beta}| = 1 \tag{3.3}$$

is satisfied.

Conformal properties of a metric space of fundamental tensor $g_{\alpha\beta}$ are those persisting under transformations leaving the null cones

$$g_{\alpha\beta}dx^\alpha dx^\beta = 0 \tag{3.4}$$

invariant. Metric tensors $g_{\alpha\beta}$ and $\bar{g}_{\alpha\beta}$ of equivalent conformal spaces are consequently related by equations of the form

$$\bar{g}_{\alpha\beta}\, d\bar{x}^\alpha d\bar{x}^\beta = \sigma(x)\, g_{\alpha\beta}\, dx^\alpha dx^\beta. \tag{3.5}$$

From (3.5) and

$$|\bar{g}_{\alpha\beta}| = \sigma^n\, |g_{\alpha\beta}|\, (x\bar{x})^2 \tag{3.6}$$

we may eliminate the factor σ, obtaining

$$\frac{\bar{g}_{\alpha\beta}}{|\bar{g}_{\mu\lambda}|^{1/n}} = (x\bar{x})^{-2/n}\, \frac{g_{\sigma\tau}}{|g_{\mu\lambda}|^{1/n}}\, \frac{\partial x^\sigma}{\partial \bar{x}^\alpha}\, \frac{\partial x^\tau}{\partial \bar{x}^\beta}. \tag{3.7}$$

Conversely if (3.7) holds the two spaces are conformally equivalent. We should observe that $g_{\alpha\beta}/|g_{\alpha\beta}|^{1/n}$ remains unaltered when $g_{\alpha\beta}$ is replaced by by $\sigma g_{\alpha\beta}$.

The conformal Riemann geometry is the invariant theory of the equations (1.1) when the conditions

$$G_{\alpha\beta} = \frac{g_{\alpha\beta}}{|g_{\mu\lambda}|^{1/n}};\ \bar{G}_{\alpha\beta} = \frac{\bar{g}_{\alpha\beta}}{|\bar{g}_{\mu\lambda}|^{1/n}};\ \kappa = -\frac{2}{n} \tag{3.8}$$

are satisfied.

In treating the conformal geometry I have been lead to a theory of $(n + 2)$ dimensional manifolds connected by non-integrable relations among the differentials of the coördinates. But my work does not appear entirely satisfactory for the invariants which arise involve arbitrary functions. Nor do the generalizations of Cartan, involving a new viewpoint of a manifold, contribute essentially to a solution of this problem.[2] We are here brought face to face with a fundamental difficulty, but it is my hope that the near future will see its satisfactory solution.

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[1] See this volume of these PROCEEDINGS, pp. 588-9.

[2] See E. Cartan, "Les espaces à connexion conforme," *Ann. de la soc. polon. de math.* (1923), pp. 171-221.

A PROPERTY WHICH CHARACTERIZES CONTINUOUS CURVES

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Let M be a point set, C_1 and C_2 mutually exclusive closed subsets of M, and K a connected subset of M such that (1) K contains no points of the set $C_1 + C_2$ and (2) both C_1 and C_2 contain limit points of K. Then K, together with its limit points in $C_1 + C_2$, will be called a set $K(C_1,C_2)M$. If M has the property that for every two of its mutually exclusive closed subsets C_1, C_2 and every connected set N containing points of both C_1 and C_2, there exists some set $K(C_1,C_2)M$ which has a point in common with N, then M will be called *normally connected.*

That not every connected point set, nor, indeed, every continuum, is normally connected, is shown by the following example: Let M be the set of points on the curve

$$y = \sin 1/x, \qquad 0 < x \leqq 1/\pi,$$

together with the set of all points on the y-axis from -1 to $+1$. Let C_1 be the set of points which M has in common with the line $y = 1$, and C_2 the set of all points on the y-axis from $-1/2$ to $+1/2$. Let N be the set of all points which M has in common with the curve $y = \sin 1/x$ together with the origin. Then N contains points of both C_1 and C_2, but there is no set $K(C_1,C_2)M$ which has points in common with N.

It is the purpose of this paper to show that the property of being normally connected characterizes those continua which are continuous curves.[1]

THEOREM 1. *A connected set which is connected im kleinen is also normally connected.*

Proof.—Let M be a connected and connected im kleinen point set, C_1 and C_2 mutually exclusive closed subsets of M, and N a connected subset of M having points in common with both C_1 and C_2. For every point x of N let $N(x)$ denote (1) in case x is not a point of the set $C_1 + C_2$, the set, F, of all points of M which lie with x in a connected subset of M that contains no point of $C_1 + C_2$, together with the set of all limit points of F

in $C_1 + C_2$, (2) in case x is a point of $C_1 + C_2$, all sets $K(C_1,C_2)M$ which contain x, or, if no such set exists, x itself.

In a paper not yet published, I have shown that if C_1 and C_2 are two closed mutually exclusive subsets of a connected, connected im kleinen point set M, there exists a set $K(C_1,C_2)M$. Hence, in case 1, there exists a set $K(x, C_1 + C_2)M$; i.e., F is not identical with x, and has at least one limit point in $C_1 + C_2$. If any F has limit points in both C_1 and C_2, it is, together with all such limit points, a set $K(C_1,C_2)M$.

Suppose that no $N(x)$ is a set $K(C_1,C_2)M$.

Divide N into two sets N_1 and N_2, such that N_i $(i = 1,2)$ contains all points x such that $N(x)$ has at least one point in common with C_i. Since N is connected, the sets N_1, N_2 are not mutually separated.[2] Hence N_2, say, contains a limit point, t, of N_1.

Suppose t is not a point of $C_1 + C_2$. As $C_1 + C_2$ is closed, there exists a circle R_1 with center at t not enclosing any point of $C_1 + C_2$. As M is connected im kleinen, there exists a circle R_2 concentric with R_1 such that every point of M interior to R_2 lies with t in a connected subset of M which lies wholly interior to R_1. As t is a limit point of N_1, there is a point s of N_1 interior to R_2, and s and t lie in a connected subset, Q, of M, such that Q lies wholly interior to R_1. Then $[N(s) - C_1 \times N(s)] + Q$ is a connected subset of M that contains t and contains no point of $C_1 + C_2$. But this implies that Q is a subset of $N(s)$ and that, therefore, t is a point of N_1. Thus a contradiction results if t is not a point of $C_1 + C_2$.

Suppose t is a point of $C_1 + C_2$, and in particular of C_2. Let T_1 be a circle with center at t and enclosing no points of C_1. As M is connected im kleinen, there exists a circle T_2 concentric with T_1 such that all points of M interior to T_2 lie with t in a connected subset R of M. As t is a limit point of N_1, there exists at least one point, u, of N_1 which lies interior to T_2, and hence belongs to R. Let

$$[N(u) + R] - N(u) = R'.$$

$N(u) + R'$ is a connected set. Hence either $N(u)$ contains a limit point of R' or vice versa. Suppose $N(u)$ contains a limit point, s, of R'. By taking a circle with center at s which encloses no points of $C_1 + C_2$ and applying the property of connectedness im kleinen of M at s, it can be shown that in this case at least one point of R' belongs to $N(u)$, which is absurd. Thus $N(u)$ cannot contain a limit point of R'. On the other hand, R' cannot contain any limit point of $N(u)$ since such a point could not be a point of C_2 and would therefore be a point of $N(u)$. A contradiction of the fact that $N(u) + R'$ is connected is therefore established, and the supposition that no $N(x)$ is a set $K(C_1,C_2)M$ is in any case contradictory.

THEOREM 2. *In order that a bounded continuum M should be a continuous curve, it is necessary and sufficient that it be normally connected.*

Proof.—That the condition stated in the theorem is necessary is a direct consequence of Theorem 1.

The condition stated in the theorem is sufficient. For, suppose M is not a continuous curve. Then there exist[3] two concentric circles, K_1 and K_2, and a sequence of subcontinua of M.

$$M_\infty, M_1, M_2, M_3, \ldots\ldots$$

such that (1) each of these sub-continua contains at least one point of K_1 and K_2, respectively, but no points exterior to K_1 or interior to K_2, (2) no two of these sub-continua have a point in common, and no two of them contain points of any connected subset of M which lies wholly in the set $K_1 + K_2 + I$ (where I is the annular domain bounded by K_1 and K_2), (3) M_∞ is the sequential limiting set of the sequence $M_1, M_2, M_3, \ldots$, (4) if H is that maximal subcontinuum of M containing M_∞ and lying wholly in $K_1 + K_2 + I$, then all of the continua $M_1, M_2, M_3, \ldots$, lie in a connected subset, Z, of $M - H$.

Let C_1 denote the set of all points common to M and the set $K_1 + K_2$. Let the radii of K_1 and K_2 be R_1 and R_2, respectively. Let R_3 and R_4 be numbers such that

$$R_1 > R_3 > R_4 > R_2,$$

and let K_3 and K_4 be circles concentric with K_1 with radii R_3 and R_4, respectively. Let the set of points $[x]$ such that x is a point of K_3, K_4, or the annular domain bounded by K_3 and K_4 be denoted by D. Let C_2 be the set of points common to H and D.

Clearly C_1 and C_2 are mutually exclusive closed sets.

Let P be a point of C_2 which is also a point of M_∞, and which lies in $D - (K_3 + K_4)$. Let F denote the set $Z + P$. Then F is a connected set containing points of both C_1 and C_2. By the condition of the theorem there is a set J which is a set $K(C_1,C_2)M$ and which has at least one point in common with F. I shall show that this is impossible.

Denote by J' the set of points obtained by adding to J its limit points. The point set J' is a subcontinuum of $K_1 + K_2 + I$, and as it contains points of C_2, and hence of H, it must be a subset of H. As $F - P$ is a subset of $M - H$, the point which F and J have in common must be P. The point P is, then, a limit point of the connected set $J - J \times C_2$, since J is a set $K(C_1,C_2)M$. Hence $[J - J \times C_2] + P$ is a connected subset of H which contains P and at least one point (a point of C_1) exterior to D. As all of the points of H in D are points of C_2, $J - J \times C_2$ must then contain points of C_2. This is absurd. Thus the supposition that M is not a continuous curve must be false, and the theorem is proved.

[1] A continuous curve is a bounded continuum which is connected im kleinen. A point set M is said to be connected im kleinen at a point P if for every circle R with center at P there exists a circle T such that all points of M interior to T lie in a connected subset of M which lies wholly interior to R. If M is connected im kleinen at every one of its points, then it is said to be connected im kleinen. See Hahn, H., *Jahresber. D. Math. Ver.*, **23**, 1914 (318–322) and *Wien. Akad. Sitz.*, **123**, Abt. IIa, 1914 (2433–2489). Also Mazurkiewicz, S., *Fund. Math.*, **1**, 1920 (166–209). Also Nalli, P., *Rend. Circ. Mat., Palermo*, **32**, 1911 (391–401).

[2] Two sets M and N are mutually separated if they are mutually exclusive and neither contains a limit point of the other.

[3] See my paper "Concerning Continuous Curves," *Fund. Math.*, **7**, 1925 (340–377). Also R. L. Moore, *Bull. Amer. Math. Soc.*, **29**, 1923 (289–302), as well as footnote (2), *Fund. Math.*, **6**, 1924, p. 210.

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*TELEPHONIC COUPLING OF ACOUSTIC AND ELECTRICAL OSCILLATIONS EVIDENCED BY THE PINHOLE PROBE**

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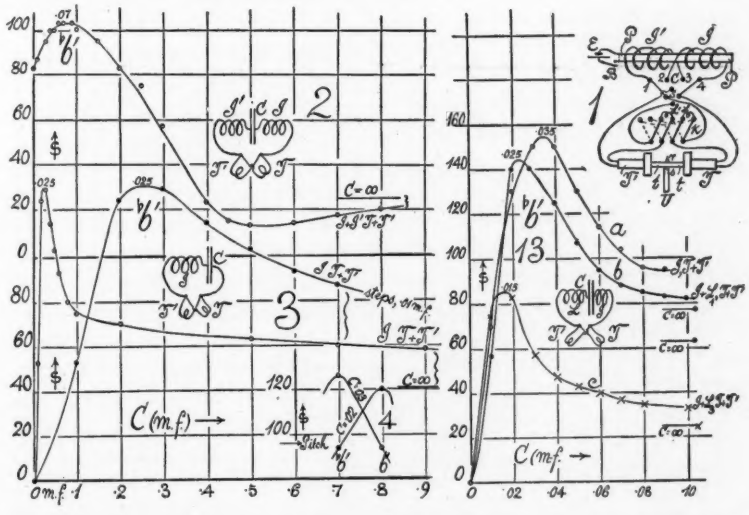
1. *Apparatus.*—Figure 1 gives a diagram of the connections. Here PP is the primary, with the electromotive force E (3 cells with 20 or more ohms resistance) and B the periodic break. The coaxial secondaries I and I' terminate in the clamps 1, 2, 3, 4, which when joined at 2, 3, produce a single secondary. Either coil may be used alone, the two joined in parallel, or even differentially, in the usual way. The parallel circuits are branched from the switch S, which admits of the reversal of the current in one of the telephones (T'). The two telephones T, T' are joined by the acoustic pipe tt with its salient and reëntrant pinhole probes s, r, and the interferometer U-gauge lies beyond U. The four terminals of these high resistance telephones end on one side of the commutator K, already described.[1] By the aid of it, the external resistances and inductances R, L, R', L', may be inserted in either one telephone or the other by swivelling the bars of K as indicated by the dotted lines. Capacities are inserted at pleasure between clamps 2, 3, as at C. The phase haromonics of the pipe tt, were ${}^{\flat}b''$, ${}^{\flat}b'\,d'$, etc.

2. *Long Range Graphs.*—A large variety of experiments were made with this apparatus, of which a few typical examples are given in the curves, figures 2 to 13. The ordinates are the fringe displacements s, varying with the displacement of mercury level in the U-gauge, and therefore with the acoustic pressure or intensity of the node at the middle of the pipe tt, figure 1.

The abscissas are the excess capacities inserted between the clamps 2, 3, and observed either in steps of 0.1 microfarad or 0.01 microfarad, as stated. The legend $I + I'$ denotes that the secondaries were used in series, $T + T'$ that the telephones were used in parallel and in phase. The inserts of the secondary circuits show how the notation is to be understood, C being the condenser. $C = \infty$ gives the fringe displacement when the condenser is short circuited. When T' alone is used the plate of T (cut out) acts as a reflector of the sound wave. The eye-location of the maximum is suggested in all curves. Owing to the large resistance in circuit (in T and T') the crests are flattish and the maximum thus admits of considerable shifting along the contours.

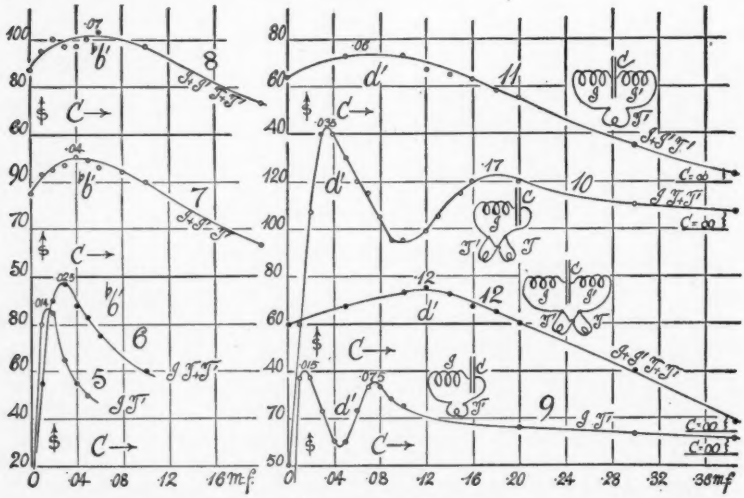


The graphs, figures 2 and 3, are examples of the behavior for a long range in C. In figure 2 the coaxial secondaries were used together, which implies an internal capacity C_1 of about 0.02 microfarad. Hence the graphs begin with a fringe displacement, s, of over 80. The maximum is tentatively located at $C = 0.07$ microfarad. The graph thereafter passes through a flat minimum and then rises again asymptotically to $C = \infty$. I was under the impression that this part of the curve might hold a maximum referable to the primary, but found no evidence.

The graph, figure 3, is given in stops of both 0.1 and (near the crest) of 0.01 microfarad. Here but one secondary (I) was used, the other being short circuited. T and T' are again in parallel. Hence these graphs begin at $s = 0$, indicating that there is no inherent or initial capacity appreciable. The maximum is now at $C = 0.025$ and is very much sharper

when compared with figure 2. Moreover the graph shows no minimum, but falls continually to the low asymptote $C = \infty$. This final s is relatively higher here, than in the preceding case.

3. *Off-tune Crests.*—Maxima are obtained when the break pitch (B figure 1) differs from the pipe pitch (tt), but not at the same pitch and capacity. This is very liable to lead to errors and the tuning must be very precise; i.e., the crest of crests must be located in C. Figure 4 gives an example in which the correct crest is at $^{b}b'$ and $C = 0.03$ microfarad; but there is a very good spurious one at pitch b' if $C = 0.02$, which if taken would constitute a large error in the results. A possible shift of crest pitch between b' and a' was recorded.



4. *Graphs Traced Near the Crests.*—The graphs for the time being are particularly interesting near the crests, as it is from these that test coefficients of induction may be computed ($1/L = \omega^2 C(1 + (d/2\pi)^2)$, from the given pitch of organ pipe and known capacity. The logarithmic decrement (d) is immaterial, so long as the crests have not been sharpened. Hence observation in steps of 0.01 microfarad are given in figures 5 to 12, from $C = 0$ through the maxima. The primary break (B) was set at $^{b}b'$ and also near d', seeing that the organ pipe harbors sharp phase nodes for each of these cases (see figure 1, volume 11, page 582 of these PROCEEDINGS). The graphs are thus divided into two groups corresponding to the $^{b}b'$ pitch and d' pitch of the break circuit in the primary. In each group the four cases designated by $(I;T')$, $(I;T+T')$, $(I+I,T')$, $(I+I',T+T')$ are worked out, with the meaning indicated by the inserts. $C = \infty$ refers to the short circuited condenser.

In the $^{\flat}b'$ group it is observed that the C-position of the $T + T'$ crests is roughly twice as large as for the T' crests. Since the telephones contain very much the greater part of the inductance of the circuit, this result for inductances in parallel is to be anticipated. In the d' group figures 11 and 12 show the same result. In figures 9 and 10 two persistent double crests are in evidence, each pair of which conforms to the rule. Figures 7, 8 also give a suggestion of double crests, but I was not able to get definite proof.

To account for the double crests it is necessary to recognize electric oscillation occurring within the intervals of the break of the primary. Thus if the latter is of permanent pitch d', the electric oscillations may be d', $^{\flat}b'$, $^{\flat}b''$, etc., depending on the capacity in circuit. For this reason, if one attempts to compute L from C and the pitch of the primary note, one reaches a curious and apparently hopeless jumble of results, in spite of the clear cut character of the graphs in figures 5 to 12. In figures 7, 8, 11, 12, moreover, there is an unknown, small but appreciable, inherent capacity of coaxial coils. In this case $(I + I')$ the maxima are flat conformally with the larger resistance.

5. *Excess Inductances without Inherent Capacity.*—To obtain information on these points, it seemed best to insert an independent excess coil of known coefficient L, as shown by the insert in figure 13, keeping L sufficiently remote from I. The coils L_1 and L_3 were used for this purpose and the three graphs of figure 13 worked out with the meaning indicated by the legends. These indicate that if the effective pitch is not $^{\flat}b'$ ($\omega^2 = 10^6 \times 8.61$) of the primary break and telephone plate, but the octave $^{\flat}b''$ ($\omega^2 = 10^6 \times 8.61 \times 4$), the results came out as reasonably as the flattish crests permit. For instance ($L_T = 1.2$, $LT + T' = 0.6$ henry, $L_I = 0.29$ henry)

Coil	$0 + I, T + T'$	$L_1 + I, T + T'$	$L_3 + I, T + T'$
Total L computed	0.83	1.16	1.94 hen.
Do.—L_I	0	0.33	1.11 hen.
Actual values	$L_I + L\ T + T' = 0.89$	$L_1 = 0.32$	$L_3 = 1.4$

Discrepancies here are referable to the eye-location of the maximum.

6. *The Effective Pitch.*—Thus the pitch at which the pipe responds is not necessarily the pitch of the telephone plate, but usually some harmonic. Using this suggestion I worked out the following adjustment of data:

	BREAK AND T PITCH, $^{\flat}b'$			
Circuit	I, T'	$I, T + T'$	$I + I', T'$	$I + I', T+T'$
Figure	5	6	7	8
Crest at C m. f.	0.017	0.025	0.04	0.07
L from C	1.71	1.16	1.93	1.29
L actual	1.5	0.9	1.9	1.3
Pitch taken	$^{\flat}b''$	$^{\flat}b''$	$^{\flat}b'$	$^{\flat}b'$

	BREAK AND T PITCH, d'					
Circuit	I, T'		$I, T+T'$		$I+I', T'$	$I+I', T+T'$
Figure	9		10		11	12
Crest at C m. f.	0.015	0.075	0.035	0.17	0.06	0.12 m. f.
L from C	1.9	1.55	0.83	0.69	1.9	1.0 hen.
L actual	1.5	1.5	0.89	0.83	1.9	1.2 hen.
Pitch taken	$^{\flat}b''$	$^{\flat}b'$	$^{\flat}b''$	$^{\flat}b'$	$^{\flat}b'$	$^{\flat}b'$

These rather crude relations could easily be improved; but they go as far as the flat crests warrant and they seem definitely to exclude any other pitch. Thus $^{\flat}b'$ occurs 6 times, $^{\flat}b''$ 4 times and d' not at all, even when this is the telephone pitch. Finally the double crests in figures 9 and 10 are the octaves $^{\flat}b''$, $^{\flat}b'$, stimulated by the d' (c.f. survey of pipe crests, l.c.).

Practically therefore a primary break of variable pitch does not seem to be needed, a spring mercury break of fixed pitch (d' for instance), sufficing. Finally the short pipes (15 cm.) harbor $^{\flat}b''$ and $^{\flat}b'$; but the multiresonance of the pipe survey below d' may be spurious.

* Advance note from a Report to the Carnegie Inst. of Washington, D. C.

[1] *Proc. Nat. Acad. Sci.*, **11**, pp. 581–584, 1925.

INTERPRETATION OF THE HYDROGEN AND HELIUM SPECTRA

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The anomalous Zeeman effect and multiplicity of spectral terms have been generally supposed to result from magnetic interaction between the valence electrons and the core of the atom. The doublet and triplet separations appear, however, to arise from relatively rather than from magnetic action. Pauli[1] has accordingly suggested that the multiplicity and related effects all result from peculiarities inherent to the valence electrons themselves, and that there is no magnetic interaction between a closed shell of electrons and valence electrons outside it. In particular, he connects the double levels in atoms having one valence electron with an assumed duality (Zweideutigkeit) in the quantum laws. If this suggestion is correct, we should expect the spectra of hydrogen and helium to resemble those of the alkalies and alkaline earths, respectively, except for shielding. It is the purpose of the present paper to show that this resemblance exists. If the evidence presented be regarded as sufficient, Pauli's suggestion receives support; we can also conclude that even hydro-

gen demands a change in the conventional quantum principles for its proper description.

Helium.—Helium and the alkaline earths resemble each other in having two series of terms, one of multiplets (doublets in He, triplets in the alkaline earths), and the other of singlets, between which combinations are possible (Lyman[2] has recently found $1S$–$2p$ in He). The normal state both in He and in the alkaline earths is a singlet S level, and in neither case is there a multiple s level of the same total quantum number as that of the normal S state. For all higher quantum numbers, the triplet s levels in the alkaline earths, and the doublet s in He, are more tightly bound than the corresponding singlet levels. The p orbits are pene-

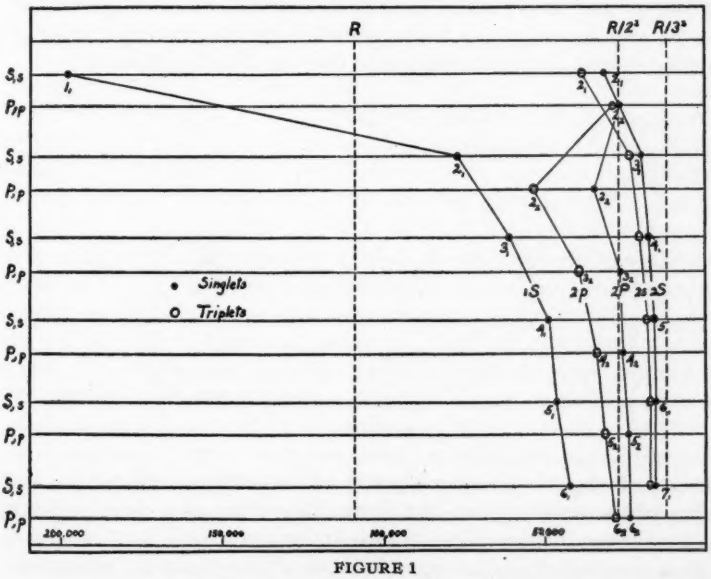


FIGURE 1

trating for all alkaline earths, non-penetrating in He. This would be expected from the reduced shielding in He, but prevents smooth extrapolation of the p levels to He. We give in figure 1 the terms of the sharp and principal series for the alkaline earths and He. (The terms for Be are estimated from the known lines of the spectrum, and should not be enough in error to affect the curves appreciably.) It is seen that the extrapolation to He is very reasonable, not only as regards the larger features but also in such details as the separation between the singlet and multiplet s levels. There are, however, two points in which He does not obviously resemble the alkaline earths: the multiplicity of the double levels, and the Zeeman effect.

Multiplicity. The first term in the principal series of r plets in He, the only resolved fine structure, is an inverted doublet in.ead of an ordinary triplet. This indicates that if He is really similar to t' .aline earths, the triplet must have degenerated into a doublet, a ıust also have become inverted. It is remarkable that the corresponding triplet of Be shows abnormalities which seem to be the beginning of such a process. In figure 2 the triplets are plotted for the alkaline earths, the magnitudes being divided by Z^2, the square of the atomic number. The division by Z^2 is used simply because it reduces all the values to the same order of magnitude (see Sommerfield, "Atombau und Spektrallinien," 4th ed., p. 666). It is seen that the Be triplet[3] is abnormally small, and that the levels p_1 and p_0 are abnormally close together, suggesting that between Be and He these two levels coalesce, and also cross p_2 and rise above it. The possible extrapolation to He is indicated in figure 2.

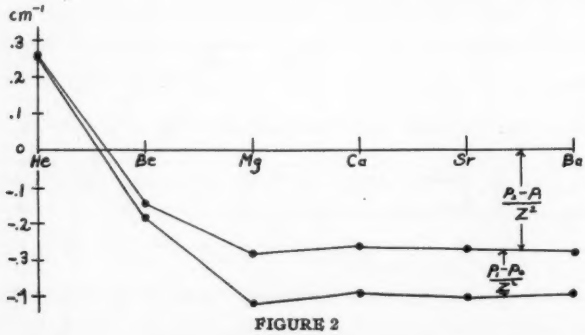


FIGURE 2

Zeeman effect. The Zeeman effect in He[4] is rather complicated. The first line of the sharp series is double in the absence of a field. At low fields the strong component splits into an approximately normal triplet, the weak component into a triplet with about twice normal separation. At higher fields, the weak line shows interaction with the strong one, one component being lost, another remaining, until the final effect is not greatly different from a single normal triplet. For the corresponding line in an alkaline earth the weak field pattern is that of an ordinary (sp) triplet. For the strong line this is $\frac{(0)\ (1)\ 2\ 3\ 4}{2}$ with $(0)/2$ and $2/2$ as the strong components; for the next line, it is $\frac{(1)\ 2\ 3}{2}$; for the final one, $(0)/2$. This shows a resemblance to what is found in He. The principal components of the resolution of the strong line form in fact a normal Zeeman triplet. It might be supposed that the weaker lines would obscure this; but in Be, Back[3] finds that the intensities are anomalous, the component 3/2 of the strongest line having an intensity only 1/10 of that

of 2/2, and the 4/2 being still fainter. This suggests that a tendency is present in Be which leads to the complete suppression of all but the normal triplet in He. For the parallel components, the agreement is not so good: in Be the component $(O)/2$ is abnormally weak, that at $(1)/2$ being strengthened, whereas in He the $(O)/2$ is the only one visible. This discrepancy, however, does not seem serious when one considers the profound change the Be triplet must undergo in degenerating into the He doublet. For the weakest line, there is in He as in Be a Zeeman triplet of twice normal separation. For strong fields, the analogy does not hold so well; both show Paschen-Back effects, but a closer resemblance can hardly be traced. Thus the evidence from Zeeman effect for our hypothesis of the similarity of He to the alkaline earths, although not particularly favorable, seems at least not adverse.

Hydrogen and Ionized Helium.—The alkalies have doublet spectra. The sharp series terms are single, all the others double. The doublet separation is thought to depend entirely on relativity,[5] the difference between the s, p, d, . . terms on the other hand coming from the different shielding of the nucleus by the electrons of the core, for the differently shaped oribits. To each term are assigned three quantum numbers, n, k and j, or, following Pauli, n, k_1, k_2, where k_1 and k_2 are used by analogy with Bohr and Coster's numbering of the X-ray levels. The sharp series terms have $k_1 = 1$, the principal series $k_1 = 2$, etc. One term of any doublet has $k_2 = k_1$, the other has $k_2 = k_1 - 1$, the term with $k_2 = k_1$ being the smaller except in inverted doublets. Then all terms with the same n and k_1 have the same shielding energy, all with the same n and k_2 the same relativity energy. Pauli's "duality" is seen in the fact that one quantum number is used to determine one kind of energy, another for the other kind. The selection principles for transitions are $k_1 \longrightarrow k_1 \pm 1$, $k_2 \longrightarrow k_2, k_2 \pm 1$. This scheme extrapolates to H simply by reducing the shielding to zero, so that all terms with given values of n and k_2 have exactly the same energy, regardless of k_1. For example, the terms 3_{22} and 3_{32} (that is, terms with $n = 3$, $k_1 = 2$ and 3, respectively, $k_2 = 2$) are equal; but they differ from terms 3_{33} by the relativity energy which, on the ordinary theory, would be the difference between 3_3 and 3_2. We discuss the spectrum of such a system, first in the absence of a magnetic field, then in the presence of such a field.

Fine structure. Evidently the set of term values is exactly the same as on the usual theory; but the quantum numbers are different, making new transitions possible and changing the intensities of the fine structure. The hydrogen fine structure is so obscured by the natural breadth of the lines that no information can be obtained from it, and we must turn to the spectrum of ionized helium. For Paschen's data the reader is referred to Sommerfeld, figures 89–92. The only measurements of value for the

present purpose are those taken when no great field is present. The line λ 4686 (4⟶3) is the most informing. It is represented schematically in figure 89b, and by an intensity curve in figure 90. The possible transitions on our scheme are shown in table 1, where we also give the transitions possible on the usual theory, and Sommerfeld's lettering of the corresponding lines. Forbidden transitions are enclosed in parentheses.

TABLE 1

FINE STRUCTURE OF λ 4686, He II

LINE	SOMMERFELD'S THEORY	PRESENT THEORY			
Ia	$4_4 \longrightarrow 3_3$	$4_{44} \longrightarrow 3_{33}$			
Ib	$(4_3 \longrightarrow 3_3)$	$(4_{33} \longrightarrow 3_{33})$,	$4_{43} \longrightarrow 3_{33}$		
Ic	$4_3 \longrightarrow 3_3$	$4_{33} \longrightarrow 3_{33}$,	$(4_{32} \longrightarrow 3_{33})$		
Id	$(4_1 \longrightarrow 3_3)$	$(4_{11} \longrightarrow 3_{33})$,	$(4_{21} \longrightarrow 3_{33})$		
IIa	$(4_4 \longrightarrow 3_2)$	$(4_{44} \longrightarrow 3_{22})$,	$(4_{44} \longrightarrow 3_{32})$		
IIb	$4_3 \longrightarrow 3_2$	$4_{33} \longrightarrow 3_{22}$,	$(4_{43} \longrightarrow 3_{23})$,	$(4_{33} \longrightarrow 3_{32})$,	$4_{43} \longrightarrow 3_{32}$
IIc	$(4_2 \longrightarrow 3_2)$	$(4_{22} \longrightarrow 3_{22})$,	$4_{32} \longrightarrow 3_{22}$,	$4_{23} \longrightarrow 3_{32}$,	$(4_{32} \longrightarrow 3_{32})$
IId	$4_1 \longrightarrow 3_2$	$4_{11} \longrightarrow 3_{22}$,	$(4_{21} \longrightarrow 3_{22})$,	$(4_{11} \longrightarrow 3_{32})$,	$4_{21} \longrightarrow 3_{32}$
IIIa	$(4_4 \longrightarrow 3_1)$	$(4_{44} \longrightarrow 3_{11})$,	$(4_{44} \longrightarrow 3_{21})$		
IIIb	$(4_3 \longrightarrow 3_1)$	$(4_{33} \longrightarrow 3_{11})$,	$(4_{43} \longrightarrow 3_{11})$,	$(4_{33} \longrightarrow 3_{21})$,	$(4_{43} \longrightarrow 3_{21})$
IIIc	$4_2 \longrightarrow 3_1$	$4_{22} \longrightarrow 3_{11}$,	$(4_{32} \longrightarrow 3_{11})$,	$(4_{23} \longrightarrow 3_{21})$,	$4_{32} \longrightarrow 3_{21}$
IIId	$(4_1 \longrightarrow 3_1)$	$(4_{11} \longrightarrow 3_{11})$,	$4_{21} \longrightarrow 3_{11}$,	$4_{11} \longrightarrow 3_{21}$,	$(4_{21} \longrightarrow 3_{21})$

The following differences are noted between the two theories: on the present theory, *Ib* is allowed, though previously forbidden; *IIc* and *IIId* are doubly allowed, though previously forbidden, *IIb*, *IId* and *IIIc* have each two possibilities of realization against one before. When we look at Sommerfeld's figure 90, we see that no check is possible in the case of *Ib*, *IIb*, *IIc* or *IId*, for they all lie in a complicated system whose principal members are *Ia* and *IIb*, and which is not well resolved. Not knowing the expected magnitude of *IIIc* exactly, this line also gives little information. The line *IIId* happens, however, to lie in a very suitable place for observation, and it is present with considerable intensity, contrary to the requirement of the conventional theory. Kramers explained its presence, with that of other forbidden lines, by assuming an external electric field. But its magnitude is very large for this; estimating its intensity by multiplying height and breadth from figure 90, we find *IIId* to be about 14 times as strong as *Id*, a forbidden line, and almost exactly half as strong as *IIIc*, an allowed line. It is interesting to note in connection with this that the principal components of *IIIc* and *IIId* are, respectively, $4_{22} \longrightarrow 3_{11}$ and $4_{21} \longrightarrow 3_{11}$, or simply the doublet of the principal series, whose intensities are known to be in the ratio 2 to 1. This apparent quantitative agreement between observation and theory should not be taken too seriously, however, both on account of the other lines $4_{32} \longrightarrow 3_{21}$ of *IIIc* and $4_{11} \longrightarrow 3_{21}$ of *IIId*, which might make the theoretical ratio different from 2 to 1, and because it is not stated whether the observations have

been corrected for the characteristic curve of the photographic plate, so that we cannot be sure that the observations represent true intensity. In the other lines measured by Paschen, $5 \longrightarrow 3$ and $6 \longrightarrow 3$, Sommerfeld gives no intensity curves, but only schematic diagrams. The lines corresponding to *IIId* are again the only ones capable of giving information. In $5 \longrightarrow 3$, figure 91, *IIIe* (analogue of *IIId*) is present with considerable intensity, although it is not as large proportionally as before; the forbidden line *Ie* (analogue of *Id*) is much weaker than *IIIe*. In $6 \longrightarrow 3$, the resolution is not good enough to make it possible to draw conclusions.

Zeeman effect: The Zeeman effect for hydrogen can hardly be resolved for weak fields. (See Sommerfeld, p. 669 for discussion and references.) For strong fields, however, an effect is observed in H_α and H_β which seems to resemble the ordinary Paschen-Back effect: the various components of the line grow together to give a single normal Zeeman triplet, the breadth of the lines being less than of the original fine structure. This phenomenon, quite inexplicable on the usual theory, is to be expected on the present interpretation. In strong magnetic fields, the level 2_{11} would split into magnetic energy levels with normal separation; 2_{22} and 2_{21} would combine into a single set of magnetic levels centering about the center of gravity of the term, or two-thirds of the distance from 2_{21} to 2_{22}. Similar phenomena would occur also for the 3 or 4 quantum levels, but on account of the smaller doublet separation the various sets of Zeeman components for these levels could be considered to coincide. Thus the structure of H_α would be a superposition of several normal triplets, the greatest spreading corresponding to approximately two-thirds of the original separation 2_{22} to 2_{21}, so that the lines would appear only about two-thirds as broad as the initial fine structure.

The ionized helium line λ 4686 has been measured accurately enough at weak fields[6] to give some information about the ordinary Zeeman effect. Hansen and Jacobsen observed that the lines *Ia* ($4_{44} \longrightarrow 3_{33}$) and *IIb* ($4_{33} \longrightarrow 3_{22}$) each split up in fields of about 5000 gauss into a normal triplet of broadened line. We should expect that at these fields the 4 quantum levels would show the Paschen-Back effect, while the 3 quantum levels would still show ordinary Zeeman effect; that is, we should have a partial Paschen-Back effect. A sodium line analogous to *IIb* is the line $3_{33} \longrightarrow 3_{22}$. A photograph of the partial Paschen-Back effect for this line is given in Back and Landé's book on Zeeman effect, figure 31 in the plate at the end. (In the description of this figure on p. 208, the theoretical separations for the two lines of the doublet seem to have been interchanged.) While a number of components are present, the intensities are such that the effect is strikingly like that of a normal triplet with broadened lines. Back and Landé give no similar photograph for the other line, $4_{44} \longrightarrow 3_{33}$; but in general the Zeeman effect

approaches the normal effect more and more closely as k_3 increases, so that this also would probably resemble a normal triplet. The patterns which we should expect for our helium line, at fields of the order of 5000 gauss, would thus be two normal triplets of broad lines, one at the position of each of the original lines, as Hansen and Jacobsen found. For higher fields, of the order of 20,000 gauss, we should expect the double level $3_{33} - 3_{32}$ to show Paschen-Back effect as in hydrogen. This again is in accordance with the observation of Hansen and Jacobsen that at these fields the parallel components of the two lines of λ 4686 coalesce. Thus the present hypothesis seems capable of explaining the magnetic behavior of both the hydrogen and the ionized helium lines.

[1] Pauli, W., Jr., *Zeit. Physik.*, **31**, 765 (1925).
[2] Lyman, T., *Astroph. J.*, **60**, 1 (1924).
[3] Back, E., *Ann. Physik.*, **70**, 347 (1923).
[4] Paschen and Back, *Ann. Physik.*, **39**, 897 (1912).
[5] Bowen and Millikan, *Physic. Rev.*, **24**, 209 (1924).
[6] Hansen and Jacobsen, *K. Danske. Vid. Sels., Mat.-Fys. Med.*, **3**, 11 (1921).

RADIATION EMITTED BY OPTICALLY EXCITED ZINC VAPOR

By J. G. Winans

University of Wisconsin

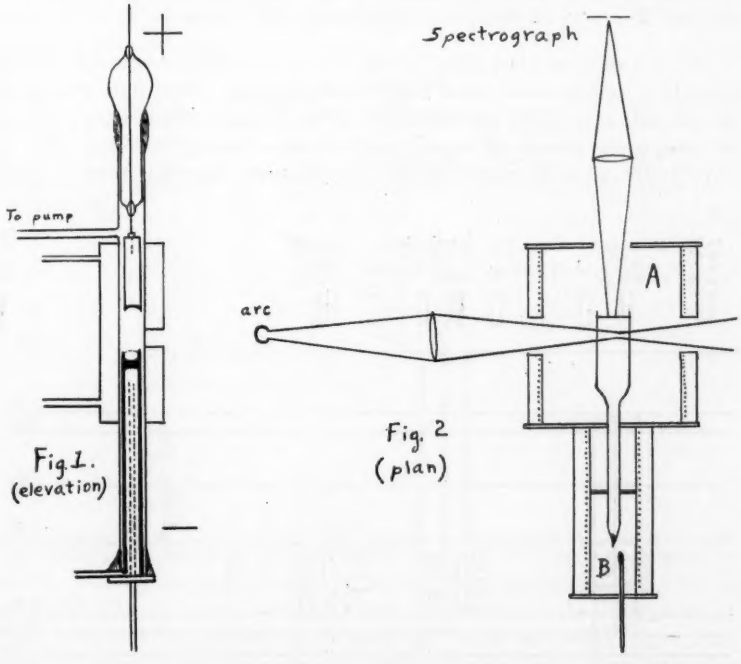
Communicated October 24, 1925

Studies of the radiation emitted by a metallic vapor when it is illuminated by radiation from a cooled arc of that metal have been made for mercury by Wood,[1] Fuchtbauer[2] and others; and for mercury, cadmium, lead, bismuth and thallium by Terenin.[3] These results serve either as a verification of the Bohr energy level scheme of an atom or as a means for identification of certain energy levels in an atom whose series relations are unknown.

The apparatus used in the present work with zinc was similar to that used by Terenin[3] though several important modifications have been made in the arc shown in figure 1. The anode is a molybdenum cylinder closed with a concave piece of molybdenum at one end and supported on a tungsten wire sealed into a pyrex tube. The cathode is an iron rod bored out to permit water cooling, the zinc being contained in a cup at the top. The anode and cathode are held in a quartz tube by sealing wax and the whole arc enclosed in a water-cooling jacket containing a hole through which the radiation is emitted. The arc is started by an induction coil and operated from 500 volt d. c. mains at a current of 3 to 4 amperes. A large inductance is connected in series and a weak magnetic field is used to con-

centrate the discharge on the side of the quartz tube next to the window. This type of arc is very satisfactory but it is necessary to clean it frequently with concentrated nitric acid and then boiling potassium hydroxide since the zinc reacts with the quartz forming a brown deposit. In part of the work the arc was operated in hydrogen at a pressure of about 0.1 mm. Under these conditions the 2138 line was very strongly absorbed.

The quartz optical system, the quartz tube containing the zinc vapor under examination (called resonance tube), and furnace are shown in figure 2. The resonance tube was evacuated, baked out, some zinc dis-



tilled in, then sealed off at a pressure of 10^{-5} mm. In operation the part *A* of the furnace was kept about 120 degrees hotter than *B* to avoid condensation of the zinc in the larger end of the resonance tube. Part *B* was kept at 360°C. for all observations. This corresponds to a zinc vapor pressure of 0.011 mm. Photographs were taken with a small quartz prism spectrograph containing lenses of focal ratio f 6.5, aperture 3.2 cms., and a 60 degree prism with a 4.7 × 4.3 cm. face. Exposures of from fifteen minutes to two hours were required.

Experimental Results.—When the resonance tube was illuminated by the

total radiation from the zinc arc the following lines appeared in the optically excited radiation.

WAVE-LENGTH	NOTATION	INTENSITY	WAVE LENGTH	NOTATION	INTENSITY
2138	$1S-2P$	9	4810	$2p_1-1s$	1
3075	$1S-2p_2$	4	4722	$2p_2-1s$	3
			4680	$2p_3-1s$	4
3345	$2p_1-3d$	10			
3302	$2p_2-3d$	8	6239 ?	$2P-3d$	1
3282	$2p_3-3d$	2	6362 ?	$2P-3D$	1
2801	$2p_1-4d$	2			
2771	$2p_2-4d$	1			
2756	$2p_3-4d$	1			

When a piece of plate glass 0.7 cm. thick was inserted between the arc and the resonance tube, these lines did not appear. This plate was found to transmit with nearly full intensity all wave-lengths longer than 3300 and to completely absorb all wave-lengths shorter than 3150. The lines at λ 6239 ($2P$–$3d$) and λ 6362 ($2P$–$3D$) are doubtful since they were obtained

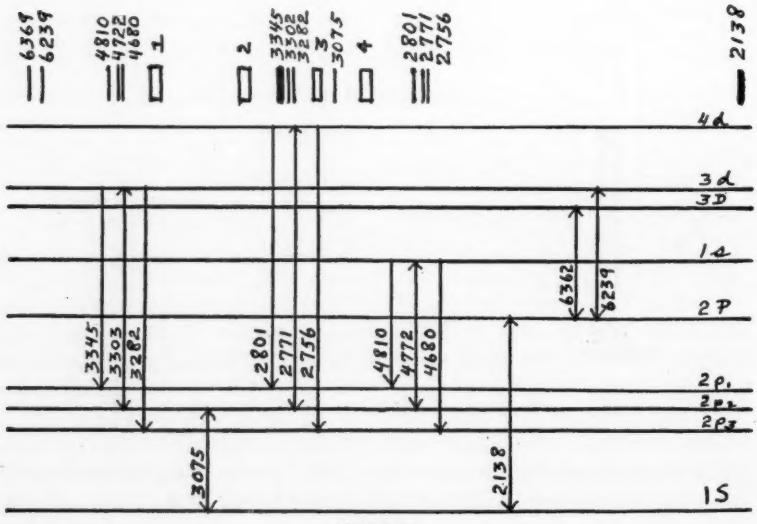


FIGURE 3

on only one plate, the arc being operated at an oil pump vacuum. Further observations will be made and the intensities of all lines will be measured.

The results obtained may be interpreted by the energy diagram of figure 3. For example consider the production of λ 3345 ($2p_1$–$3d$). An electron in the $1S$ level of a zinc atom is raised to $2p_2$ by absorption of λ 3075

($1S$–$2p_2$). From $2p_2$ it is raised to $3d$ by absorption of λ 3303 ($2p_2$–$3d$). From $3d$ it returns to $2p_1$ emitting λ 3345 ($2p_1$–$3d$). The production of the remaining lines may be worked out in the same way, there being two possible methods to obtain λ 6239 ($2P$–$3d$).

Optically Excited Band Spectra.—In addition to the above lines four bands were observed in the spectrum of the optically excited radiation. At the corresponding positions in the arc spectrum, four groups of bands diffused toward the violet were observed. The wave-lengths of the heads of these bands in the arc were measured with a one meter concave grating (15,000 lines per inch) using zinc lines as standards. Some of these bands have been observed by others and have been attributed to zinc. The wave-lengths are as follows.

		WINANS		HARTLEY AND RAMAGE[4]	HAGENBACH AND SCHUMACHER[5]	MORSE[6]	HOWSON[7]
		ARC IN HYDROGEN		OXY-HYDROGEN FLAME	ELECTRODELESS RING DISCHARGE	ELECTROLYTIC INTERRUPTER ZN WIRE IN HYDROCHLORIC ACID	IRON TO BRASS ARC IN A VACUUM
GROUP	INT.		INT. (ARC)				
1	5	4301.5	9	4302	4300.5	4299.0	4300.959
		4240.9	7	4240	4240.0	4237.8	4239.315
		3576.7	5				
2	3	3536.7	4		3534.6		
		3499.8	2		3499.3		
		3158.8	4				
3	4	3135.7	3				
		3116.3	2				
		3103.5	1				
		2976.3	1				
4	2	2961.6	3				
		2952.5	3				

This table gives the wave-lengths of the heads of the bands in Ångström units. The column under Int. gives the intensities in the optically excited radiation. The intensity of these bands compared to the series lines is much greater when the arc is operated in hydrogen. Under these conditions the bands are especially strong in the optically excited radiation making it possible to observe groups 2, 3 and 4. The individual members in each group were not resolved in the optically excited radiation. These bands constitute all or part of what Sommerfeld has called a "band system." The two bands at λ 2961.6 and λ 2952.2 probably belong to some other band system since their character is quite different and their intensity relative to the neighboring band λ 2976.3 is not in accordance with the sequence of intensities in the other groups.

The intensity of these band groups relative to the series lines in the optically excited radiation is much greater than in the arc radiation. If they

were due to scattered light the relative intensity would be sma[illegible]ther experiments with improved apparatus are in progress.

Bands similar to group 1 have been observed in the spec[illegible] of cadmium[4,7] but Terenin[3] did not mention having observed them in the optically excited radiation of cadmium vapor. The reproduction of his arc spectrum did not show these bands to be present, probably because his arc was not as effectively cooled as that used in this work and was not operated in hydrogen.

Impact Fluorescence.—One resonance tube used in this work contained mercury as an impurity. When this tube was illuminated by the water-cooled quartz mercury arc the mercury lines which were observed by Wood,[1] the λ 3075 ($1S$–$2p_2$) zinc line, and a strong line at λ 3267 ± 6 were obtained in the optically excited radiation. The appearance of the zinc line is explained by what has been called "sensitized fluorescence," but which is perhaps better described as "impact fluorescence." A normal zinc atom is excited by impact with an excited mercury atom, the mercury atom becoming normal. The excited mercury atom possesses more energy than is required to excite a normal zinc atom. The extra energy is changed into increased kinetic energy of the two colliding atoms. Impact fluorescence has been observed and interpreted for several metals by Cario and Franck,[8] Kopferman[9] and Donat.[10] The strong line at λ 3267 ± 6 was undoubtedly the λ 3261 ($1S$–$2p_2$) resonance line of cadmium excited by impact of a normal cadmium atom with an excited mercury atom showing that cadmium was present as an impurity in the zinc. Further observations of this effect will be made using higher temperatures and greater mercury vapor pressures.

In conclusion I wish to express my thanks to Dr. C. E. Mendenhall who suggested the problem and directed the work.

[1] Wood, R. W., *Proc. Roy. Soc.*, **106A**, 1924 (679–694).
[3] Fuchtbauer, *Phys. Zeit.*, **21**, 1920 (635–638).
[3] Terenin, A., *Zeit. Phys.*, **31**, 1925 (26–49).
[4] Hartley and Ramage, *Sci. Trans. Roy. Soc., Dublin*, **7**, 1901 (339).
[5] Hagenbach, A. and Schumacher, H., *Zeit. Wiss. Phot.*, **19**, 1919 (142–148).
[6] Morse, H. W., *Astrophys. J.*, **19**, 1904 (177).
[7] Howson, E. E., *Astrophys. J.*, **36**, 1912 (290–292).
[8] Cario, G. and Franck, J., *Zeit. Phys.*, **17**, 1923 (202).
[9] Kopferman, H., *Zeit. Phys.*, **21**, 1924 (316–325).
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ON CONTACT RECTIFICATION BY METALLIC GERMANIUM

By Ernest Merritt

Department of Physics, Cornell University

In a recent note[1] I have called attention to the significance of the phenomena of contact rectification in their bearing upon theories of thermo-electric power and have suggested that because of the relative simplicity of the conditions the study of these phenomena in the case of the elements is especially desirable. Experiments with selenium contacts were described in the September number of these Proceedings. The search for elements showing rectifying properties has been continued and in the present communication I shall describe the measurements made thus far with metallic germanium.

The high thermo-electric power of germanium;[2] the fact that in Mendeleef's series Ge falls in the same group as Si, which is one of the best rectifying materials known; and the further fact that Ge and Si have the same crystal form, all lead us to expect that germanium contacts will show rectifying properties. This expectation has been confirmed by tests made with a specimen of this rare metal weighing twelve grams which was loaned to me by Professor L. M. Dennis. The rectifying action with Ge is less marked than with many other materials, but the contacts are quite stable and the behavior at different points on the surface is more uniform than in the case of most crystal rectifiers. Except in certain unusual cases mentioned below the resistance of a Ge contact is least for a current flowing into the Ge from the contact point. This is also the direction of the rectified current when an alternating e.m.f. is used. It will be noticed that the direction of rectification is opposite to that found for Se.

In experiments on rectifying contacts it is, of course, necessary to make electrical connection with the sensitive material at two points. In the case of crystal detectors it is customary to mount the crystal in such a way as to make one of the contacts of relatively large area, the tacit assumption being that if the area of contact is sufficiently great the current density will be so small that any rectifying action at this large contact will be negligible. A common procedure is to mount the specimen in melted solder or Wood's metal. With most crystals the matter is simplified by the fact that the rectifying or detecting action is confined to certain sensitive spots, while the greater part of the surface is relatively inactive. With Ge, however, practically all parts of the surface seems to be active, so that it is particularly important that the area of contact with the support should be large. Since it did not seem desirable to use solder or Wood's metal the specimen was clamped between two iron jaws, the latter being covered with several layers of tin foil, or of iron or copper

gauze to give a yielding surface and a larger contact area. Until this procedure was adopted erratic results were obtained which are probably to be ascribed to imperfect connection between the specimen and its support.

The resistance of the contacts used was sufficiently low so that the fall of potential through the measuring instrument was appreciable. To simplify the procedure a Weston voltmeter placed in series with the contact was used as an ammeter, with a reversing key so located as to reverse the current through the contact but not through the voltmeter. The reading of this instrument gave at once the correction to be applied to the applied e.m.f. (as measured by a second voltmeter) while the reading divided by the resistance of the instrument gave the current through the contact. The current sensitivity was adjusted to any desired value by means of shunts.

The contacts to be used were made by pressing the rounded end (radius of curvature about 0.5 mm.) of various metal rods against the Ge surface by means of a spring. To avoid disturbances due to vibration the whole apparatus was mounted on a block of wood that was supported by stretched rubber bands. In the case of all the results here recorded contact was made with a part of the Ge surface that had been polished with emery.

Characteristic curves for several typical contacts are shown in figure 1. These were all taken with the Ge surface freshly polished. With the applied e.m.f. less than 3 volts the contacts were stable and the curves could be reproduced. It will be noticed that the asymmetry of the characteristics is not nearly so marked as in the case of Se[1] or of many of the crystals used in crystal detectors.

Contacts at different points on the Ge surface did not give identical characteristics. But the differences were usually not great; and the change resulting from the use of a different metal for the contact point often seemed to be greater than that due to a change in the position of the contact. The curves plotted in figure 1 have been selected as representing in each case the average behavior of contacts with the metal named. The characteristics for Pt and Cu contacts fall so close to that for Fe that they have not been plotted. Al and Zn also gave nearly identical characteristics. In fact all five of these metals were so nearly alike in their behavior that their relative position on the plot is probably not significant.

There can scarcely be any doubt, however, that a real difference exists between the behavior of Bi and Sb. The contacts of these two metals for which characteristics are shown in figure 1 differed greatly in resistance. But although the bismuth contacts almost always had the higher resistance, I do not regard this difference as of great significance, since the resistance of a contact depends so greatly upon the pressure and the area. The bismuth contacts varied in resistance more than in the case

of any of the other metals studied and some bismuth contacts had resistances as small as that of the antimony contacts. The most significant difference between the two metals was in the *form* of the characteristic. The antimony characteristic in figure 1, which is typical of all those observed, shows a marked difference between the positive and negative branches, while all of the bismuth characteristics that I have observed have their positive and negative branches nearly alike. In other words antimony gives better rectifying contacts than bismuth.

The anomalous behavior of Bi contacts was more marked when the surface of the Ge was slightly tarnished, as was always the case after heating. Contacts with a tarnished surface always showed a much higher resistance than contacts with a freshly cleaned surface. With

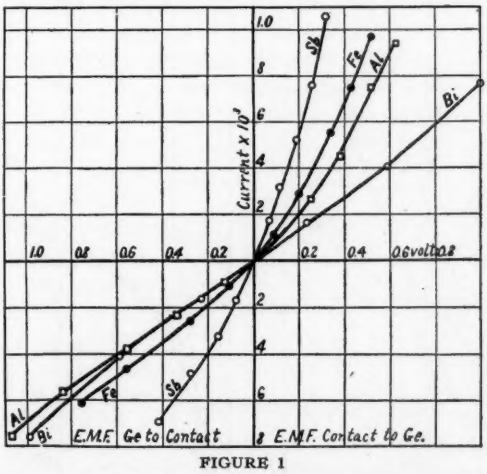

FIGURE 1

Current-voltage characteristics for contacts between *Ge* and *Bi*, *Al*, *Fe* and *Sb*.

Bi the two branches of the characteristic were nearly alike, although the current through the contact from Bi to Ge was usually greater than the current for the same e.m.f. reversed. But contacts were frequently found for which the current was greater in the direction Ge to Bi. With an alternating e.m.f., Bi contacts usually gave normal rectification, i.e., the rectified current flowed from Bi to Ge. But many contacts were found which gave no rectification, while a few gave rectification in the opposite sense. In the case of such contacts the direction of the current developed by warming the Bi contact was also reversed, being, in each case, opposite to the rectified current. Some contacts were found which rectified normally for light pressure but in which the rectified current

was reversed when the pressure was increased. In these contacts the direction of the thermo-electric effect was normal and was not reversed by pressure.

Selenium contacts sometimes show a similar reversal of rectification. The development of a thermal e.m.f. because of heating at the contact offers a plausible explanation of such reversals. But the magnitude of the reversed effect raises some doubt as to the correctness of this explanation. Further study of these anomalous effects is much to be desired but is made difficult by the instability of the contacts that show such effects.

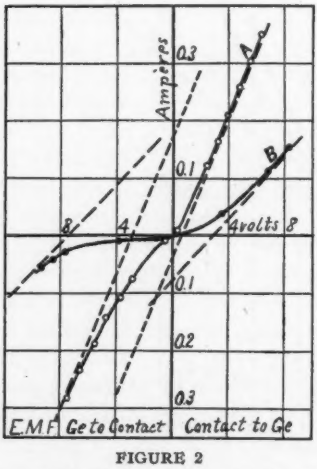

FIGURE 2

A. Contact between *Ge* and *Sb*. B. Contact between *Ge* and steel.

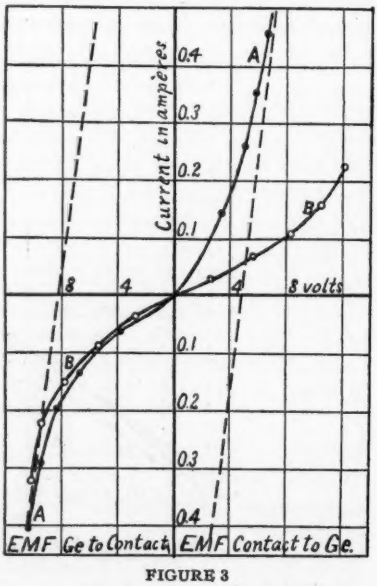

FIGURE 3

Contacts between polished *Ge* and different corners of a broken piece of *Ge*.

It seems probable that the relatively small rectifying action of bismuth contacts and the occasional reversal of the direction of the rectified current and of the thermo-electric effect are connected with the fact that Bi lies much closer to Ge in the thermo-electric series than any of the other metals used. The results thus indicate that the connection between thermo-electric power and rectifying action is extremely close.

With Ge contacts, as in the case of Se contacts, each branch of the characteristic approaches a linear asymptote for large values of the applied e.m.f. In the case of the negative branch this is often not at all obvious,

for the contacts are even more likely to break down at high e.m.f's. than are Se contacts and it is rarely possible to follow the negative branch far enough to make sure that it approaches an asymptote. Curves for two unusually stable contacts are shown in figure 2. It will be noticed that the two asymptotes are quite accurately parallel, as was found to be the case with Se contacts. Curve A of figure 3 also shows parallel asymptotes.

The two characteristics shown in figure 3 are for contacts between polished Ge and two different corners of a broken piece of the same metal. While curve A shows as great a difference between its two branches as either of the curves in figure 1, curve B is nearly symmetrical. The fact that the characteristic is curved indicates, however, that for either direction of current there is a marked deviation from Ohm's law. The non-linear character of the characteristics is probably in all cases more significant than the fact that the two branches are usually unlike.

Bidwell[2] has found that the thermo-electric power of Ge against Pt reverses sign at about 300°C. Since the direction of the rectified current in contact detectors is usually opposite to the current produced by heating it seemed of interest to study the behavior of Ge contacts at temperatures above the reversal point, the expectation being that the rectification would also be reversed. The specimen was therefore placed in a furnace and characteristics were determined at different temperatures as the furnace was heated and then allowed to cool. A copper-advance thermo-element was used in measuring the temperature; since no special precautions were taken to secure high accuracy errors of as much as 10°C. are not unlikely. A series of characteristics determined in this way is shown in figure 4.

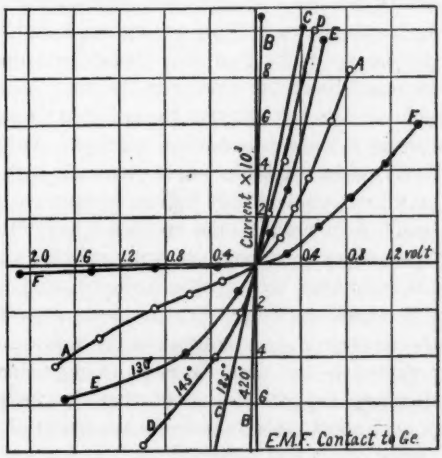

FIGURE 4

Characteristics for a *Ge-Fe* contract at different temperatures. A before heating. F after cooling.

It will be noticed that the resistance of the contact is greatly reduced at high temperatures. At 400°C. the resistance, which was of the order 200 ohms at ordinary temperatures, had fallen to about 5 ohms. For temperatures up to 200°C. the resistance was least for the direction

contact to Ge—i.e., when electrons passed out of the Ge at the contact. But the asymmetry of the characteristic became less as the temperature was raised. At 185°C. some difference was still observable, the resistance being 39 ohms in one direction and 45 ohms in the other. But at 220°C. when the resistance had fallen to 5 ohms, and for higher temperatures, no difference between the two directions could be detected. In the hope that rectifying action might be detected by a more sensitive measuring instrument the voltmeter was replaced by a galvanometer and an alternating e.m.f. of about 1 volt was used. At temperatures above 200°C., the galvanometer deflections were small and erratic; if any rectification occurred it was so small as to be entirely masked by the uncertainties of measurement. But upon cooling below 200°C., the rectifying action increased with great rapidity; a change of 10° in the temperature of the contact caused the galvanometer deflection to increase from 1 division or less to more than 150.

The resistance of the contact after cooling was always considerably greater, and the asymmetry of the characteristic more marked, than before heating. A difference between the original and final characteristic such as is shown in figure 4 in curves A and F was found in all cases. There can be no doubt that this change was due to the formation of a film, presumably of the oxide, on the Ge. The presence of such a film was indicated by the fact that the polished Ge surface usually showed a slightly tarnished appearance after heating. Although too thin to show interference colors this film was of extremely high resistance. When the contact point was pressed only lightly against such a film-covered surface—e.g., by its own weight—the resistance was often as great as ten megohms. Upon using more pressure or by rubbing the contact point back and forth the resistance was greatly reduced, and by repolishing the Ge surface the conditions before heating were restored. The rapid diminution in the resistance of the contact as the temperature is raised is what would be expected if the film consisted of an oxide of germanium. The results therefore support the view[3] that the presence of a high resistance film is an important and possibly an essential factor in determining the behavior of contact rectifiers.

[1] Merritt, E. *Proc. Nat. Acad. Sci.*, **11**, 572, 1925.

[2] Bidwell, C. C. *Physic Rev.*, **19**, 447, 1922.

[3] Indications that a film separates the two surfaces in a rectifying contact have been found by numerous observers. Perhaps the most direct evidence of the importance of such a film is to be found in the papers of Goddard and Flowers here referred to: Goddard, R. H., *Physic Rev.*, **34**, p. 423, 1912, and Flowers, A. E., *Ibid.*, **29**, 445, 1909.

THE ORIGIN OF SPECIES. II

DISTINCTIONS BETWEEN RECTIGRADATIONS AND ALLOMETRONS

BY HENRY FAIRFIELD OSBORN

AMERICAN MUSEUM OF NATURAL HISTORY, NEW YORK

Read before the Academy November 11, 1925

In this second contribution on the origin of species I may again quote the geneticist, William Bateson, 1921: "That particular and essential bit of the theory of evolution which is concerned with the origin and nature of *species* remains utterly mysterious." It is important to recall that in 1889 appeared my "The Palaeontological Evidence for the Transmission of Acquired Characters" (Osborn Bib., 1889, 46), containing my early observations on *continuity* in the origin of species, and that in 1894 appeared Bateson's "Materials for the Study of Evolution," setting forth his theory of *discontinuity* in the origin of species—a wholly antithetic theory. At the time I wrote my paper I was inclined to the Lamarckian interpretation, namely, that continuity in the origin and evolution of mechanical adaptations might be due to continuity of adaptive modifications in the soma. But I gradually abandoned the Lamarckian interpretation, because while apparently the right one in a few adaptations it is flatly contradicted in the majority of adaptations.

Thus in 1890 I abandoned theoretic interpretations of every kind and devoted myself to the most laborious and intensive observations on the origin of species, first in the primates, second in the lower ungulates, third in the higher ungulates. Then I sought closely consecutive ascending series of higher ungulates and observed that the delusive *discontinuity* which exists between contemporary zoölogical species is replaced by actual *continuity* when we observe the ascending ancestral lines leading to existing species. Thus in support of prior observations by invertebrate palaeontologists, we are able to affirm as the result of thousands of observations based on lines of ascent of many different kinds, that the *origin of species in all the bio-mechanical characters we have observed is an absolutely continuous process.*

In this respect the evolution or phylogeny of each specific character is exactly like the development or ontogeny of each specific character. This I set forth in my first contribution to the Academy as III—the Principle of Continuity.

I shall now explain in more detail VI—the Principle of Allometry (i.e., the evolution of alloiometrons = allometrons), also VII—the Principle of Rectigradation in mechanically adaptive organs.

While it has long been known that species originate either through the origin of new characters (neomorphs) or through changes in the proportion

of inherited characters (palaeomorphs), it was observed in palaeontologic ascending series that *the origin of new specific characters follows a different course from the origin of new specific proportions.* These differences were summed up by myself in 1914 as follows:

Ten Principles of VII—Rectigradation.[1]—(1) Rectigradations are strictly new *localized* outgrowths (e.g., hypocone, horn rudiment, mesostyle); (2) rectigradations are *neomorphs*, new outgrowths, numerically new characters (e.g., a tooth 4 to 6 cusps); (3) rectigradations arise chiefly through *ancestral affinity* (e.g., Perissodactyla evolve many similar "rectigradations" in their molar and premolar teeth which never appear in Amblypoda or Artiodactyla); (4) rectigradations apparently may have no adaptive value at the start or origin; (5) the origin of *similar rectigradations* in independent lines *always* gives rise to parallelism or convergence (e.g., Orohippus molar parallels that of Hyopsodus and of Notharctus in the independent origin of hypocone, conules); (6) rectigradations give rise to certain so-called "homologies" (e.g., the mesostyle of Mesohippus and of Palaeosyops is said to be "homologous" although one is not descended from the other or even from a common ancestor; the common ancestor had no "mesostyle"); (7) rectigradations are not known to be produced experimentally in ontogeny; (8) rectigradations are neomorphs, relatively rare phenomena or infrequent; (9) rectigradations may be measured by indices and ratios (i.e., as allometrons), as soon as they attain sufficient size (e.g., horns and cusps); (10) rectigradations characteristic of one phylum tend to appear in a rudimentary form and at a late period in another phylum as "homologies" (e.g., "crista," "crochet," "antecrochet" in horses, titanotheres, rhinoceroses).

Nine Principles of VI—Allometry.[2]—(1) Allometrons are general changes of proportion, not localized (e.g., dolichocephaly, brachycephaly, hypsodonty, brachyodonty, dolichodonty); (2) allometrons are allomorphs or heteromorphs, or new changes of proportion, quantitatively new characters (e.g., a broad tooth becomes a narrow tooth); (3) allometrons may be totally independent of ancestral affinity (e.g., Perissodactyla and Amblypoda may both become dolichocephalic); (4) allometrons apparently may have *adaptive significance* throughout (e.g., changes of limb proportion); (5) the origin of allometrons in animals of the same ancestry may be either convergent or divergent; (6) allometrons give rise only to *analogies*, never to "homologies" in any sense (e.g., dolichocephaly elongated cranium, in *Homo* and in *Menodus*); (7) allometrons may be produced experimentally in ontogeny (e.g., dog with fore limbs removed develops new proportions of hind limbs); (8) allometrons or allomorphs are universal generic and specific phenomena (see Miller's "Catalogue.. Mammals ..Western Europe," Brit. Mus., 1912); (9) allometrons constantly afford indices and ratios.

DISSENTING OPINIONS

(*a*) At the time these contrasts between rectigradations and allometrons were formulated, my colleague, William King Gregory, drew different conclusions (May 14, 1914) from my observations, as follows: "(1) Rectigradations and allometrons are intergrading phenomena, different manifestations of similar causes; (2) they are both caenotelic characters which gradually develop out of palaeotelic or stem characters; (3) allometrons are rigidly correlated with other changes; they are also widespread correlations; (4) rectigradations are focal outgrowths, partly independent proliferations, somewhat after the fashion of cancer cells and like them usually arising at foci of stimulation; (5) the frequent parallelism of rectigradations in related phyla results from the fact that the far greater number of palaeotelic characters (both in the adult and in the germ-cell) rigidly restricts the possible avenues of further differentiation along lines already entered upon (e.g., aquatic animals excluded from cursorial adaptations, fossorial animals from volant adaptations, etc.). Hence all ungulates of many orders develop similar rectigradations and similar allometrons; (6) the same germinal mechanism serves for the transmission and differentiation both of rectigradations and of allometrons—whatever general hereditary factors result in evolution are equally responsible for both classes of phenomena."

(*b*) Another of my colleagues, William Diller Matthew, also dissented (May, 1914) from the conclusions drawn from my observations: "As for the difference between rectigradations and allometrons, that seems to me rather descriptive than real. It is precisely the difference that Cope attempted to draw between generic and specific characters. The one was a question of the appearance or disappearance of a specified character, the other a difference in degree of development or proportion. Such a distinction is valid (whether or not it be desirable to use it) for descriptive and taxonomic purposes. That it corresponds to any fundamental difference in the real nature of the changes I do not see. In my experience on Carnivora and other phyla, *all changes whether of degree or kind arise out of individual variations of small magnitude, fixed and emphasized through selection* so far as I can judge, and guided by the adaptive trend of the phylum. Mendelian inheritance, I suspect, is responsible for the complete elimination of many unused fluctuative variations in a phylum, e.g., the parastyle of p^4 in one group of Fissipedia, while in another group it is emphasized by its utilization. In the Miacidae it is a fluctuating character except in one group where it has been seized upon for development."

(*c*) Gregory's phrase, "Rectigradations are focal outgrowths, a partly independent proliferation, somewhat after the fashion of cancer cells

and like them usually arising at foci of stimulation," is a Lamarckian interpretation similar to that of Cope. On the other hand, Matthew's phrase, "In my experience on Carnivora and other phyla, all changes whether of degree or kind arise out of individual variations of small magnitude, fixed and emphasized through selection so far as I can judge, and guided by the adaptive trend of the phylum," is a purely Darwinian interpretation.

In my fourth contribution on the origin of species, to be presented to the Academy at the April meeting, I shall continue with a summary of concrete observations made between 1894 and 1925, which I believe substantiate my interpretations of 1914.

[1] This term, signifying advance in adaptive direction, was first used by Osborn in "Evolution of Mammalian Molar Teeth" (1907) "to refer to the origin of new cusps or cuspules which appear determinately, definitely, orthogenetically in both the upper and lower teeth—quite *independently* in different orders of mammals, and separated perhaps by vast intervals of time."

[2] This is a convenient abbreviated derivative of the Greek *αλλοιοσ, different,* and *μετρον, that by which anything is measured.*

VITAL STATISTICS OF THE NATIONAL ACADEMY OF SCIENCES. I. AGE AT ELECTION[1]

By Raymond Pearl

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Communicated November 9, 1925

I. The constitution of the Academy provides (Article IV, Section 5) that the total membership shall not exceed 250. The present membership is not far from this limiting figure. The mortality rate is less than 15 per year, and if elections go on at that rate, as they have done for a number of years prior to the last annual meeting, it must soon result that the present constitutional upper limit to membership will be reached. Taking cognizance of this fact the Academy, at its autumn meeting in Boston in 1924, ordered by resolution that an existing committee on nomination and election, of which the writer chanced to be chairman, should make a comprehensive study of the biostatistical situation which its membership presents and report the results, in order to form a factual basis for such discussion or action as might seem desirable regarding the problem. The present series of papers is the response to this order of the Academy.

II. The first step in this study was the making of a list of all the persons who had ever accepted election to the Academy, from its inauguration

on March 3, 1863, up to May 1, 1925, which date was taken as the terminal point of the present series of statistical studies. This means that the persons elected to membership at the annual meeting of 1925 are not included in any of the statistics in this and the following papers. For all of the persons falling in the defined universe the following data were set down:

1. Date of birth.
2. Date of election to the Academy.
3. Age at election, in years and tenths of a year. In figuring these ages the date of the annual meeting was assumed always to have been April 15, except in the case of the charter members, where the actual date March 3, was used. The amount of labor involved in looking up the precise date of each annual meeting was too great to be undertaken. The error made in choosing a fixed date, April 15, is insignificant, so far as concerns any conclusions which will be drawn in this series of papers. Actually it is probably a fair *average*, for it appears that while in recent years the annual meetings have tended to be late in April, there was a tendency in earlier years to hold the meetings at earlier dates.
4. Date of death (if dead).
5. Age of death (if dead), or age on May 1, 1925 (if still living). These ages were recorded in years and tenths of years. They were figured exactly, allowing for the extra day in leap-years. With some trouble the exact date of birth of every member was found, and the date of death of the deceased. In using the term "exact" I mean only that a published record of what purported to be the dates in question was in every case finally found. In figuring tenths of years under this heading, and under 3 above, a year was taken to include 365 days.
6. Duration of service in the Academy, in years and tenths of years.

Space cannot be taken to publish the complete list of names, with the above data. Typewritten copies, however, will be filed with the Home Secretary.[2]

III. The first subject chosen for discussion is that of age at election. Table 1 gives the data. In this table the class unit of age is taken as two years. The total interval is divided into four groups: (*a*) the original charter members, (*b*) those elected in the years 1864 to 1883 inclusive, (*c*) those elected in the years 1884 to 1904 inclusive, and (*d*) those elected in the years 1905 to 1924 inclusive. The means and standard deviations in age at election, calculated from the distributions of table 1, are set forth in table 2.

From these tables the following deductions seem warranted:

1. As was to be expected in the nature of the case the original membership was of a high average age at election. There was great variation in this age, but only slight skewness in the distribution, the median age differ-

ing by less than six months from the mean age. All these relations are to be expected in the initial composition of a body like an academy. Of the 48 charter members 4, or 8 per cent, were under 38 years of age at the time of election.

TABLE 1

AGE AT ELECTION TO THE NATIONAL ACADEMY OF SCIENCES

AGE IN YEARS	NUMBER ELECTED IN PERIOD				
	1863 (CHARTER)	1864–1883	1884–1904	1905–1924	1863–1924
26.0–27.9	..	1	..	..	1
28.0–29.9	1	..	..	..	1
30.0–31.9	..	3	1	..	4
32.0–33.9	1	5	1	..	7
34.0–35.9	..	9	4	2	15
36.0–37.9	2	17	4	6	29
38.0–39.9	4	8	3	10	25
40.0–41.9	2	7	8	12	29
42.0–43.9	4	4	6	14	28
44.0–45.9	..	6	10	23	39
46.0–47.9	4	3	7	18	32
48.0–49.9	2	4	..	29	35
50.0–51.9	6	9	5	18	38
52.0–53.9	4	4	3	16	27
54.0–55.9	3	..	5	18	26
56.0–57.9	4	2	..	11	17
58.0–59.9	2	3	3	9	17
60.0–61.9	1	1	1	4	7
62.0–63.9	1	4	2	9	16
64.0–65.9	2	2	..	2	6
66.0–67.9	1	..	..	5	6
68.0–69.9	1	1	2	3	7
70.0–71.9	..	1	..	2	3
72.0–73.9	1	1	..	1	3
74.0–75.9	1	..	..	1	2
76.0–77.9	..	..	..	..	..
78.0–79.9	..	..	..	..	..
80.0–81.9	..	..	..	..	..
82.0–83.9	1	..	..	..	1
Totals	48	95	65	213	421

TABLE 2

BIOMETRIC CONSTANTS FOR AGE AT ELECTION TO THE NATIONAL ACADEMY

PERIOD	MEAN	MEDIAN	STANDARD DEVIATION
Charter–1863	51.71±1.08 years	51.33±1.35 years	11.10±0.76 years
1864–1883	44.47±0.71	41.29±0.89	10.22±0.50
1884–1904	46.54±0.71	45.10±0.89	8.45±0.50
1905–1924	50.54±0.37	49.48±0.46	8.09±0.26
Total, 1863–1924	48.69±0.31	48.03±0.39	9.45±0.22

2. In the first third of the Academy's existence after its initial year (the period 1864–83) election was at the rate of 4.75 persons per year over

the 20-year period. The mean age at election, 44.47 years, was lower than it has ever been since. The median age was only 41.29 years, 3 years lower than the mean age, indicating a marked skewness in the distribution. Of the 95 persons elected in this period, 35, or 37 per cent, were under 38 years of age at the time of election.

3. The second third of the Academy's life (1884–1904) included a period of extreme conservatism in respect of elections. The rate of election over this 21-year period was only 3.1 persons per year. The mean age at election was 46.54 years, approximately two years higher than in the earlier period. The variability in age at election had become less, the standard deviation being nearly 20 per cent smaller than in the earlier period. Also the distribution was less skew. Of the 65 persons elected in the period, 10, or about 15 per cent, were under 38 years of age at the time of election.

4. In the last 20 years (1905–1924) the rate of election has been 10.65 persons per year. This enormous increase over the previous practice of the Academy has presumably been due to the operation of a rule for a number of years which virtually compelled the election of 15 per annum. During this period the average age at election has been much higher than ever before in the Academy's history (except among charter members, who may be expected *a priori* to show a high mean age at election), 4 years higher than in the previous period, and 6 years higher than in the first 20 years of the Academy's history. The variability in age at election is also lower than ever before. Of the 213 persons elected in this period only 8, or 3.8 per cent, were under 38 years of age at time of election. During the first 20 years of the Academy's existence there were elected nearly 10 times as many young men, in proportion to the total absolute number elected, defining a "young man" by such a generous limit as 38 years.

All of the statistical relations regarding age at election which have been set forth above suggest an ever increasing conservatism on the part of the Academy in respect of election, save only in the matter of annual rate under pressure of an arbitrary rule which virtually forced the election of 15.

The more important of the relations brought out in tables 1 and 2 are shown graphically in figure 1.

The following table, derived from data given by Schuster[3] gives the same constants as table 2, for Fellows of the Royal Society, over periods nearly, though not exactly, comparable in time.

TABLE 3

BIOMETRIC CONSTANTS FOR AGE AT ELECTION TO THE ROYAL SOCIETY

PERIOD	MEAN	MEDIAN	STANDARD DEVIATION	PER CENT UNDER 35 YEARS OF AGE AT ELECTION	PER CENT UNDER 40 YEARS OF AGE AT ELECTION
1848–1873	44.38±0.36	43.08±0.45	10.18±0.25	19.6	38.6
1873–1897	45.75±0.38	44.10±0.48	11.05±0.27	16.3	36.0
1898–1922	46.18±0.31	45.38±0.39	8.99±0.22	9.6	27.2

The same tendency to an increase in average age at election is observable in the Royal Society as has manifested itself in the National Academy,

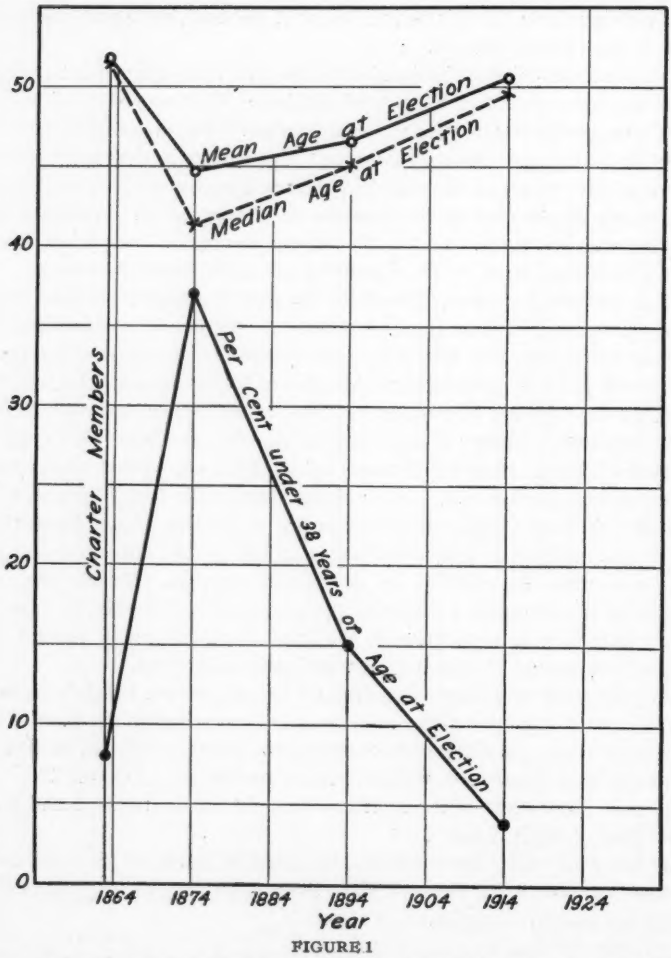

FIGURE 1

Showing the mean and median ages at election and the percentage of those elected who were under 38 years of age at the time, for (*a*) charter members, and (*b*) three approximately equal periods in the subsequent history of the Academy.

but is much less marked in amount. Comparing the last period it is seen that the Royal Society elected in the period at an average age more than 4

years younger than the Academy. The variability in age at elections is about the same in the two bodies. The percentage of young men elected is generally somewhat higher in the Royal Society than in the Academy.

[1] From the Institute for Biological Research, Johns Hopkins University.

[2] All of the data comprised in this list have been twice independently checked and computed, once by Miss Helen Trybulowski and once by Dr. John Rice Miner, to whom I am greatly indebted for this service, as well as for arithmetic work on this and the following papers in this series. It is believed that they are accurate. Much help, for which I am very grateful, was obtained on different points regarding dates, etc., from Mr. Paul Brockett, Assistant Secretary of the Academy. Professor E. B. Wilson, who had been making some similar studies to these regarding the Academy membership, very kindly placed his raw data on library cards at my disposal. My list had already been completed when this material reached me, but it proved useful in checking.

[3] Schuster, A., *Proc. Roy. Soc.*, **A107**, pp. 360–376, 1925.

VITAL STATISTICS OF THE NATIONAL ACADEMY OF SCIENCES. II. ELECTIONS OF YOUNG MEN[1]

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Communicated November 9, 1925

I. In the preceding paper of this series, attention was called to the fact that the percentage of men elected at younger ages than 38 has steadily and markedly declined throughout the history of the Academy. In the whole period of the Academy's life, 57 persons (including 4 charter members) falling within the defined age category have been elected. It seems desirable to make a somewhat more detailed examination of the elections at early ages. To this end there are collected in the present paper the data regarding the tenth of the whole Academy, including both the dead and the living, which was most youthful in years at the time of election. The division point in age which most nearly (indeed almost exactly) divides the total membership into a youngest 10 per cent, and an older 90 per cent, is at 37 years. In the whole history of the Academy 43 persons, including 2 charter members, have been under 37 years of age at the time of their election. Table 1 gives certain data about these persons, likely to be of interest to the student of the history of sciences, of human biology and perhaps others.

Making the rough division into biological and physical sciences, including the one paleontologist with biology, including mathematics with the physical sciences, and leaving out of account the one economist as not amenable to such a classification, it appears that 30 of the persons elected to the Academy before reaching the age of 37 cultivated some one or another of the physical sciences, and only 12 the biological. Since the ratio of total physical elections to total biological elections is not so high as this it fol-

lows that notable achievement is attained, by those who ever attain it, somewhat earlier in the physical sciences than in the biological. This same preponderance of the physical sciences in the election of young men is noted by Schuster[2] in the Royal Society.

TABLE 1

PERSONS ELECTED TO THE NATIONAL ACADEMY OF SCIENCES AT AGES UNDER 37

YEAR OF ELECTION	NAME	AGE AT ELECTION	SUBJECT
1863	Newton, H. A.	33.0	Astronomy
1863	Rogers, F.	29.3	Engineering
1865	Mitchell, S. W.	36.2	Medicine
1865	Rood, O. N.	34.2	Physics
1865	Watson, J. C.	30.2	Astronomy
1866	Agassiz, A.	30.3	Zoölogy
1866	Johnson, S. W.	35.8	Chemistry
1868	Brush, G. J.	36.4	Mineralogy
1868	Stimpson, W.	36.2	Zoölogy
1869	Newcomb, S.	34.1	Astronomy
1872	Cope, E. D.	31.7	Paleontology
1872	Crafts, J. N.	33.1	Physics
1872	Mayer, A. M.	35.4	Physics
1872	Packard, A. S.	33.2	Zoölogy
1872	Pumpelly, R.	34.6	Geology
1872	Verrill, A. E.	33.2	Zoölogy
1873	Gill, T. N.	36.1	Zoölogy
1873	Pickering, E. C.	26.8	Astronomy
1874	Hill, G. W.	36.1	Mathematics
1876	King, C.	34.3	Geology
1876	Peirce, C. S. S.	36.6	Mathematics
1877	Coues, E.	34.6	Zoölogy
1878	Trowbridge, J.	34.7	Physics
1879	Farlow, W. G.	34.3	Botany
1881	Rowland, H. A.	32.4	Physics
1882	Remsen, I.	36.2	Chemistry
1883	Bell, A. G.	36.1	Physics
1883	Hill, H. B.	34.0	Chemistry
1883	Jackson, C. L.	36.1	Chemistry
1884	Brooks, W. K.	36.1	Zoölogy
1884	Dana, E. S.	34.4	Mineralogy
1888	Michelson, A. A.	35.3	Physics
1889	Michael, A.	35.7	Chemistry
1890	Mayo-Smith, R.	36.2	Economics
1890	Chittenden, R. H.	34.2	Physiology
1892	Barus, C.	36.2	Physics
1899	Richards, T. N.	31.2	Chemistry
1902	Hale, G. E.	33.8	Astronomy
1916	Pearl, R.	36.9	Biology
1918	Birkhoff, G. D.	34.1	Mathematics
1918	Bridgman, P. W.	36.0	Physics
1923	Webster, D. L.	34.5	Physics
1924	Compton, K. T.	36.6	Physics

Taking the particular branches of science separately gives the following frequencies:

Physics.....	11	Engineering	1 each
Zoölogy.....	7	Medicine	
Chemistry.....	6	Paleontology	
Astronomy....	5	Botany	
Mathematics..	3	Economics	
Geology.....	2	Physiology	
Mineralogy....	2	Biology	

II. The time relations of the election of young men are shown graphically in figure 1.

The contrast between the left and the right hand halves of this diagram is striking. It emphasizes the suggestion made in the first paper in this series, that the Academy has become more conservative as it has grown older, thus following a well known law of human behavior. But more will be said on this point below. In 1918 there was a very hopeful spurt in the physical sciences, when two men under 37 were taken in in one year. But the shock was severe. No young man was again taken in until five years thereafter. In the 34 years from 1891 to 1924 inclusive, only one worker in any of the biological sciences under 37 at the time of election has been admitted. Only 5 biologists under that age at election have been admitted since 1876.

III. The youngest man ever elected to the Academy was the astronomer Pickering, and the next youngest the engineer Fairman Rogers, who was a charter member. These two are the only persons ever elected at less than 30 years of age. The third youngest at the time of election was the astronomer J. C. Watson, and the fourth youngest was Alexander Agassiz, long

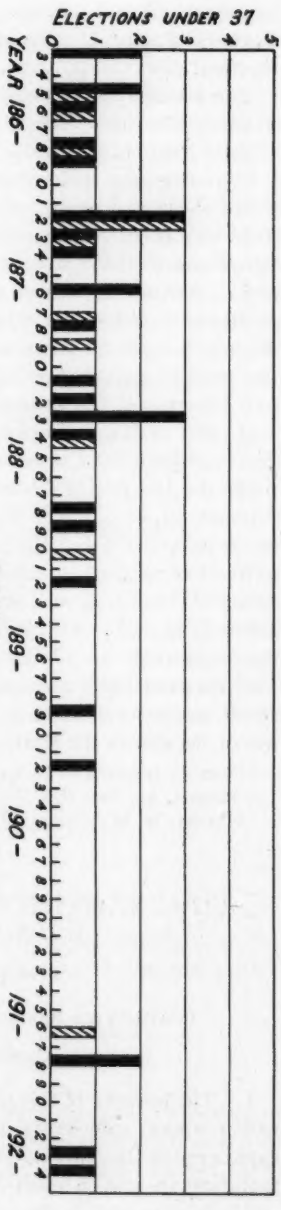

FIGURE 1
Showing the years of election of persons under 38 years of age at election. Solid black bars denote the physical sciences, and cross-hatched bars the biological.

president of the Academy. Seven of the biologists, Agassiz, Cope, Coues, Farlow, Packard, Chittenden and Verrill were under 35 at the time of their election.

The average age of election of the 12 biologists in the group was 34.42 years, while that of the 30 followers of the physical sciences was very slightly less, 34.24 years.

IV. It is easy to attribute the changing habits of the Academy relative to the election of young men to a growing conservatism of that body itself. That this is the sole cause I doubt. It is at least possible, and I incline personally to think it probable, that the increasing organization, standardization, mechanization and constant striving for efficient mediocrity in all our academic life, which every thoughtful person has seen going on during the past 30 years, and which some have deplored and vainly endeavored to stop, is showing as one of its most dreadful effects the curbing and fettering of the progress of the really brilliant student. That some such idea as this is forcing itself upon many minds is indicated by the National Research Council's study of the problem of the brilliant student, under the direction of Professor C. E. Seashore and by Professor Wheeler's[3] brilliant paper on "The Dry-Rot of Our Academic Biology." Much ink has been spilled about the subject of academic freedom, but nearly always with reference to the freedom of the professors to do various things, sometimes obviously absurd or ridiculous. Is it not about time to consider seriously the subject of the freedom, within academic precincts, of the student to develop his intellectual powers in the way he personally wants to? Perhaps some slight concessions in this direction would in time have as one result, among many other desirable ones, some lowering of the average age of election to the National Academy of Sciences.

[1] From the Institute for Biological Research of the Johns Hopkins University.

[2] Schuster, A., *Proc. Roy. Soc.*, **A107**, 360–376, 1925.

[3] Wheeler, W. M., *Science*, **57**, 61–71, 1923.

VITAL STATISTICS OF THE NATIONAL ACADEMY OF SCIENCES. III. MORTALITY[1]

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Communicated November 9, 1925

I. The purpose of this paper is to present an abridged, but for all practical purposes sufficiently detailed, life table derived from the mortality experience of the National Academy of Sciences from its foundation to, and including, the person selected at the annual meeting in 1924. The following methods were used in the computation of this table:

The total membership, both living and dead, was first classified according to age, the ages of the living being taken as of May 1, 1925, and those of the dead at death. The making of this classification has been fully discussed in the first of this series of papers. In the case of members who resigned from the Academy at some date after their election the whole exposure to risk from election to death was included in the computation, on the ground that resignation from the Academy could not be presumed to have sensibly impaired or enhanced the person's chance of survival, and on the further ground that resignation is not *per se* an event of present interest.

Central death rates for the quinquennial age groups 40–44, 45–49, etc., were calculated in the following manner: the number dying in the given age group made up the numerator. The denominator of the rate fraction, that is the person-years exposed to risk of dying was formed, e.g., for the group 40–44, by taking,

$$\begin{aligned}
&\tfrac{1}{2}\text{ (number living + number dead aged 40)}\\
+&1\tfrac{1}{2}\text{ (number living + number dead aged 41)}\\
+&2\tfrac{1}{2}\text{ (number living + number dead aged 42)}\\
+&3\tfrac{1}{2}\text{ (number living + number dead aged 43)}\\
+&4\tfrac{1}{2}\text{ (number living + number dead aged 44)}\\
+&5\quad\text{(number living + number dead aged 45 and over).}
\end{aligned}$$

This gives the total number of years lived in the age group 40–44 by persons elected to the Academy. Many of the elections, however, took place after the person had passed 40 years of age, so that a part of this number cannot properly be included in the exposed to risk of dying while a member of the Academy. There was therefore subtracted from the total years lived in a given age group the years lived within that group before election, e.g., for the group 40–44,

$$\begin{aligned}
&\tfrac{1}{2}\text{ (number elected at age 40)}\\
+&1\tfrac{1}{2}\text{ (number elected at age 41)}\\
+&2\tfrac{1}{2}\text{ (number elected at age 42)}\\
+&3\tfrac{1}{2}\text{ (number elected at age 43)}\\
+&4\tfrac{1}{2}\text{ (number elected at age 44)}\\
+&5\quad\text{(number elected at age 45 and over)}
\end{aligned}$$

and similarly for the other age groups. The difference between the two figures is the exposed to risk. The q_x's were calculated from the central death rates by the usual formula

$$q_x = \frac{2m_x}{2 + m_x}.$$

Rates were not calculated for ages lower than 40 years owing to paucity of material, there having been in the whole history of the Academy only two members dead below that age. The rates at the periods 40–44 and 85–89 are not very reliable owing to the same cause.

The q_x values were fitted by least squares with the following curve:

$$\log 1000\, q_x = 0.7679 + 0.1552x + 0.0035x^2.$$

In fitting the 40–44 value was not used, and the 50–54 and 55–59 values were smoothed before fitting.

From these fitted values of q_x the expectation of life at 5-year intervals was calculated by a modification of Hayward's short method.[2] The p_x for the middle year of each quinquennial group was taken as the mean p_x for the group and applied 5 times to the l_x of the beginning of the group. Using logarithms this is much simpler than calculating l_x for each year and quite sufficiently accurate for such material as the present. The L_x values were obtained from the approximate formula

$$L = \frac{l_x + l_{x+5}}{2} \times 5.$$

except for the 75–79 group, in which,

$$L_x = \frac{l_x + 2\, l_{x+2.5} + l_{x+5}}{4} \times 5.$$

The T_x column is obtained by adding the L_x column from below up. As has already been mentioned the q_x for 85–89 is unreliable, and comparison with the life table for white males in cities of the Original Registration States for 1901 shows that extrapolation of the fitted parabola would give probably too high values of q_x at the highest ages. The T_{80} value is therefore obtained by multiplying 37,386, the T_{80} value for the white urban males by $\frac{22{,}466}{7825}$ the ratio of the l_{80} values in the two tables. The expectation of life, $\mathring{e}_x = T_x/l_x$.

II. Table 1 presents the life table computed as described above. The meaning of the figures in the several columns is as follows: 1. Age group, needs no explanation. 2. Person-years exposed to risk of dying in each of the age groups of column 1. 3. Deaths, being the observed mortality frequency of academicians in the several age groups. 4. m_x, the raw central death rates for each group. 5. q_x, the life table death rates calculated from the m_x's as explained above. 6. l_x, number living at the beginning of each age group, out of 100,000 starting together at age 40. 7. L_x, number of the stationary life table population living in each age group. 8. T_x, the number of persons in the stationary life table population living at age x and over. 9. $\mathring{e}_x$, the expectation of life, or mean after life time, at the beginning of each age group. 10. For comparison, $\mathring{e}_x$ from Glover's[3] life tables for white males in cities of the Original Registration States in 1901, this being the nearest point to the average date of death of academicians for which a comparable general population life table is available.

TABLE 1

A LIFE TABLE FOR MEMBERS OF THE NATIONAL ACADEMY OF SCIENCES, BASED ON THE MORTALITY EXPERIENCE FROM ITS FOUNDATION IN 1863 TO, AND INCLUDING, 1924

AGE GROUP	EXPOSED TO RISK	DEATHS	m_x	q_x OBSERVED	q_x FITTED	l_x	L_x	T_x	$\mathring{e}_x$	$\mathring{e}_x$ WHITE MALES IN CITIES O. R. S. 1901
40–44	547.5	3	0.0055	0.0055	0.00586	100,000	492,763	3,018,039	30.18	25.06
45–49	856.0	8	0.0093	0.0093	0.00845	97,105	475,437	2,525,276	26.01	21.78
50–54	1085.5	15	0.0138	0.0137	0.01237	93,070	451,310	2,049,839	22.02	18.56
55–59	1123.5	14	0.0125	0.0124	0.01840	87,454	417,880	1,598,529	18.28	15.51
60–64	982.0	25	0.0255	0.0252	0.02784	79,698	372,257	1,180,649	14.81	12.80
65–69	771.0	32	0.0415	0.0407	0.04279	69,205	312,045	808,392	11.68	10.32
70–74	516.5	43	0.0833	0.0800	0.06682	55,613	237,397	496,347	8.93	8.20
75–79	262.0	34	0.1298	0.1219	0.10607	39,356	151,613	258,950	6.58	6.31
80–84	104.0	17	0.1635	0.1511	0.17108	22,466	...	107,337	4.78	4.78
85–89	22.0	7	0.3182	0.2745	0.28041	...	...	.....	...	...
90–94	3.5	...	.....	.....	.....	...	...	.....	...	...
Totals	6273.5	198	.....	.....	.....	...	...	.....	...	...

The expectation of life columns at the end of the table are shown graphically in figure 1.

III. From table 1 and figure 1 it is apparent that the members of the Academy have enjoyed a more favorable rate of mortality than that experienced by nearest comparable moiety of the general population in 1901. At age 40 the mean after lifetime of academicians has been above 5 years greater than that of urban white males. This superiority in life expectation is maintained, in diminishing but still definite degree, up to 85 years of age, and probably even to higher ages but the Academy's experience has

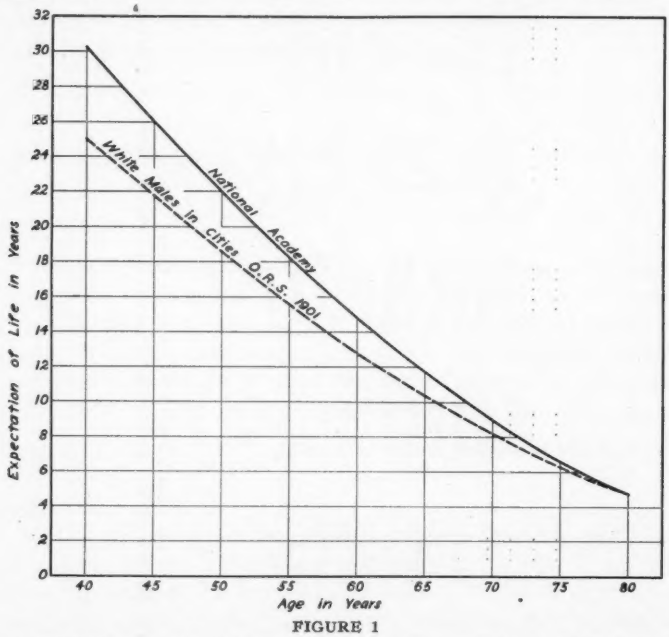

FIGURE 1

Expectations of life at indicated ages of (*a*) members of the National Academy of Sciences, and (*b*) the general population of white males living in cities in the Original Registration States in 1901.

been too meagre to make the point certain for ages beyond 75. The life table furnishes some foundation for the apocryphal remark that members of the Academy seem less busily occupied in writing each other's obituaries than might have been expected. The academicians show almost the same expectations of life as white males in the rural part of the Original Registration States in 1901, the differences amounting only to fractions of a year in favor of the rural population, being certainly smaller than the errors of sampling involved in the comparison.

The result shown in table 1 is what any actuary would have expected, because members of the Academy belong generally to occupational groups which furnish superior risks from a life insurance standpoint. The members of the Academy show a superior expectation of life along with such groups as the clergy and college professors generally, and for precisely the same reason. Election to the Academy is not to be thought of as biologically rejuvenating the elected or slowing his rate of senescence. The result merely means that persons of the kind elected to the Academy would live longer, on the average, than *l'homme moyen* of the general population, whether made academicians or not.

Schuster,[4] using a very approximate method which perhaps somewhat exaggerates the effect, finds that "the average life of Fellows of the Royal Society is, on the average, nearly six years longer than that given in the Tables (for the general population) which have been used."

[1] Papers from the Institute for Biological Research, Johns Hopkins University.

[2] Hayward, T. E., *J. Roy. Stat. Soc.*, **62**, 443–483, 1899; *Ibid.*, 683–702, 1899; *Ibid.*, **63**, 625–636, 1900; *Ibid.*, **64**, 636–641, 1901; *Ibid.*, **65**, 354–358, 1902; *Ibid.*, 680–684, 1902.

[3] Glover, J. W., United States Life Tables, 1890, 1901, 1910 and 1901–1910, Washington (Govt. Printing Office), 1921.

[4] Schuster, A., *Proc. Roy. Soc.*, **A107**, 368–376, 1925.

VITAL STATISTICS OF THE NATIONAL ACADEMY OF SCIENCES. IV. THE PRESENT LIMITATION TO TOTAL MEMBERSHIP AND OTHER MATTERS[1]

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Communicated November 9, 1925

I. Article IV, Section 5, of the Constitution of the Academy provides that the total membership of the Academy shall not exceed 250. Given the rates of mortality set forth in the preceding paper in this series, and a body of living members having the age constitution which the membership of the Academy has, when may it be expected that this upper limit of total membership will be reached, provided the Academy exercises annually its privilege of electing 15 new members? To give a formally *exact* answer to this question would involve a very elaborate series of computations, which would be more intricate than the basic data as to mortality warrant. But an approximate, and for practical purposes sufficiently close, answer can be given, and it is the first purpose of the present paper to do this.

Table 1 shows the annual mortality to be expected on the following assumptions: (*a*) that the age constitution of the living membership is as it

was on May 1, 1925, and (*b*) that the rates of mortality given in the life table of the Academy members presented in the preceding paper continue to operate. Neither of these assumptions will be strictly true over any considerable period, but both will be approximately true for the next few years. The mean age of the living membership of the Academy is not stabilized, but slowly rising all the time. The specific death rates are perhaps very slowly falling, as the general population death rates in the same period of life are by some supposed to be. But over a few years table 1 will give a sufficiently accurate indication of what is likely to happen. It should be noted that the q_x values in this table outside the range of those given in the life table in the preceding paper are, in the case of the age group 35–39 extrapolated, and in the case of the two end values from 85 on, taken from Glover's table for white males in cities, Original Registration States, 1901.

TABLE 1

EXPECTED ABSOLUTE ANNUAL MORTALITY IN THE ACADEMY

AGE	NUMBER OF LIVING MEMBERS n	q_x	nq_x
35–39	3	0.00413	0.012
40–44	7	0.00586	0.041
45–49	24	0.00845	0.203
50–54	35	0.01237	0.433
55–59	43	0.01840	0.791
60–64	33	0.02784	0.919
65–69	36	0.04279	1.540
70–74	19	0.06682	1.270
75–79	13	0.10607	1.379
80–84	4	0.17108	0.684
85–89	3	0.23205	0.696
90–94	1	0.31455	0.315
Total	221	.....	8.283

It appears from table 1 that the expected absolute mortality is approximately $8^1/_4$ persons per year. This value is almost certainly somewhat too high, because of the q_x values used beyond 85. Probably even after allowing for these it is still somewhat too high. The paucity of the experience makes all the rates above age 70 none too reliable. The academician's expectation of life, in short, is probably even better than his life table indicates at ages beyond 65 or 70. It will be conservative to take 8 as a round figure for expected deaths in a year, at the present size and state of the Academy. In the 1923 Report (dated 1924) 8 members were reported as dying in the preceding year. In the 1923–24 Report (dated 1925) the deaths of 6 members were reported, but between the time of sending this report to the press and May 1, 1925, there were 5 more deaths.

Now with a living membership of 221, an average of 8 deaths a year, and 15 elections a year, in not more than 4 years forward from May 1, 1925, it may reasonably be expected that the Academy will be very close to, or at,

the limit of 250 members. The limit may even be reached in 3 years from the date named. The mortality will then permit the election, on the average, of somewhere around 6 to 8 persons per year to replace those who have died in the year. Of course if the Academy elects fewer than 15 per year, as it did in 1925, then the period of approach to the constitutional upper limit of membership will be prolonged.

Incidentally table 1 presents the data from which may easily be deduced the biometric constants regarding the age of the living members of the Academy. These are:

Mean age of living members	$= 60.74 \pm 0.47$
Median	$= 59.83 \pm 0.59$
Standard deviation	$= 10.43 \pm 0.33$

It is seen that the present living membership is quite symmetrically grouped in age about an average of 60.74 years. The average age of the living members is approximately $11^1/_4$ years higher than the average age at election in recent times.

II. The last fact brought out suggests that an examination be made of the average number of years which an elected person serves the Academy as a member. The raw data for the elucidation of this point are presented in table 2.

TABLE 2

YEARS OF SERVICE AS MEMBERS OF THE ACADEMY

YEARS	a. MEMBERS LIVING MAY 1, 1925	b. DEAD MEMBERS	YEARS	a. MEMBERS LIVING MAY 1, 1925	b. DEAD MEMBERS
0– 1.9	15	11	28.0–29.9	3	9
2.0– 3.9	28	9	30.0–31.9	2	5
4.0– 5.9	29	13	32.0–33.9	1	6
6.0– 7.9	28	12	34.0–35.9	1	6
8.0– 9.9	21	9	36.0–37.9	3	4
10.0–11.9	17	15	38.0–39.9	–	3
12.0–13.9	14	12	40.0–41.9	2	6
14.0–15.9	13	5	42.0–43.9	2	3
16.0–17.9	14	10	44.0–45.9	–	6
18.0–19.9	2	8	46.0–47.9	–	–
20.0–21.9	7	14	48.0–49.9	1	1
22.0–23.9	6	9	50.0–51.9	1	1
24.0–25.9	5	14	52.0–53.9	2	–
26.0–27.9	4	9	Totals	221	200

On account of the fineness of the grouping and the small total number, these distributions are somewhat irregular.

The constants calculated from the distributions of table 2 are exhibited in table 3.

TABLE 3

BIOMETRIC CONSTANTS FOR DURATION OF SERVICE AS A MEMBER OF THE ACADEMY

CONSTANT	*a.* MEMBERS LIVING MAY 1, 1925	*b.* DEAD MEMBERS
Mean	12.17 ± 0.48 year	19.57 ± 0.60
Median	9.00 ± 0.60	19.00 ± 0.75
Standard deviation	10.49 ± 0.34	12.49 ± 0.42

The members who have died served the Academy before doing so an average of approximately a half year less than 20 years. Possibly someone may wonder why this figure is not larger, to correspond with the $\mathring{e}_x$ for the average age of election, as set forth in the life table in the preceding paper in the series. It ought not so to correspond, of course, because we are here dealing only with the dead members of the Academy, which group includes, *ipso facto*, all those members who died relatively young and does not include all those who attained great longevity, because some of the latter are still living. As the Academy itself grows older and accumulates a larger store of mortality experience the two sets of figures will come nearer together. With a stabilized population, in which the age specific death rates and distribution of ages at election were both constant (i.e., exhibited no secular change over a period of 100 years at least) the two values would come very close together.

As is to be expected the average length of service of the living members is less than that of the dead members, and the distribution is much more skew.

[1] From the Institute for Biological Research, Johns Hopkins University.

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*RESEARCHES ON THIAZOLES. X. THE SYNTHESIS OF SOME 2-PHENYL-BENZOTHIAZOLE ARSONIC ACIDS**

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ORGANIC LABORATORIES COLUMBIA UNIVERSITY

Communicated November 30, 1925

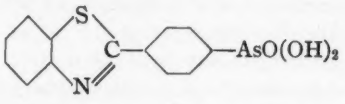
Introductory.—The great stability of the 2-phenyl-benzothiazole nucleus led us to undertake the synthesis and study of some of its arsonic acid derivatives, in the hope of discovering products of therapeutic value.

Investigation of arsenic derivatives of the benzene series has shown the therapeutic value of the NH_2 and OH groups, especially when in *o*-position to each other, and that CH_3, halogen, COOH or SO_3H, are frequently dystherapeutic in effect.

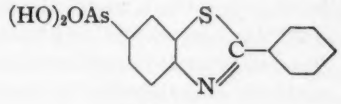
In addition to arsonic acids of 2-phenyl-benzothiazole itself, we have prepared therefore arsonic acids of its amino and hydroxy derivatives.

These arsonic acids were obtained generally from the amines through the diazo reaction, following the investigations of Bart.[1]

2-Phenyl-benzothiazole *p*-arsonic acid (I), prepared from the amine, was subsequently nitrated and the nitro reduced to the amino group. Since Fourneau[2] recently has called attention to the fact that acetylation of a *p*-amino group occasionally increases the therapeutic index, the study of the acetyl derivative is under way, as well as the condensation with chloroacetamide for the purpose of obtaining the analog of Tryparsamide. 2-Phenyl-benzothiazole-6-arsonic acid (II) was also obtained from the appropriate amine.



(I)



(II)

By nitrating the 2-(*p*-hydroxyphenyl)-benzothiazole, reducing the nitro group and diazotizing the amine so formed, the 2-(*p*-hydroxyphenyl)-benzothiazole arsonic acid was produced. The location of the nitro and amino groups in these arsonic acids has not been determined.

Preliminary pharmacological experiments with some of these new arsenic derivatives have been so promising that the investigation is being continued and expanded.

While this work was in progress, Hunter[3] reported the preparation of derivatives of 6-methyl-2-(*p*-aminophenyl)-benzothiazole (dehydrothio-*p*-toluidine) by fusing it with arsenic acid at 190–200° and also by treating it with arsenic trichloride, but as he fails to give analyses, physical constants, or other proof of the nature of his products, and as his initial material was admittedly highly impure, it is impossible to tell what he had in hand. Our own experience in fusing 6-amino-2-phenyl-benzothiazole with arsenic acid, in a manner similar to that used for the preparation of arsanilic acid, was unsatisfactory and the method was abandoned.

EXPERIMENTAL PART

2-Phenyl-benzothiazole-6-arsonic Acid.—A solution of 5.6 g. of 6-amino-2-phenyl-benzothiazole in 40 g. of concentrated sulphuric acid was poured upon cracked ice and then diazotized with 2 g. of sodium nitrite in 50 cc. of water. After adding 59 g. of sodium acetate, the mixture was poured very slowly into a suspension of 4.1 g. of potassium arsenite, 1 g. of potassium hydroxide and 2 g. of finely divided copper bronze, in 50 cc. of water, cooled to 0°. The alkalinity of the solution was maintained by the addition of more potassium hydroxide if necessary. The reaction proceeded with liberation of nitrogen. After standing for two hours at low

temperature, the mixture was boiled until the evolution of nitrogen ceased. It was then filtered, the filtrate acidified, the yellow flocculent precipitate (yield, 5.4%) collected, dried and crystallized from alcohol. Clusters of yellow needles were obtained, difficultly soluble in alcohol, and which were unmelted at 310°.

Anal. Calc. for $C_{13}H_{10}O_3NAsS$: C, 46.57; H, 2.98. Found: C, 45.62; H, 2.64.

Other catalysts were tried in this reaction, such as cuprous chloride, cement copper made by the action of zinc dust upon a copper sulphate solution, and others, but copper bronze proved most satisfactory, for its use in alkaline solution meant the prompt removal of all copper on filtration.

2-Phenyl-benzothiazole-p-arsonic Acid.—A suspension of 4.5 g. of 2-(*p*-aminophenyl)-benzothiazole in dilute hydrochloric acid (10 cc. of concentrated acid and 90 cc. of water) was diazotized at low temperature with 1.7 g. of sodium nitrite in 50 cc. of water. The soluble diazonium chloride was filtered from unchanged amine and added to a solution of potassium arsenite (from 4 g. of arsenic trioxide, 2.8 g. of potassium hydroxide and 200 cc. of water) containing copper bronze (previously washed with carbon tetrachloride and ether) in suspension, and in the presence of sufficient potassium hydroxide to keep the mixture alkaline. After standing for some time at room temperature, it was heated at 90° until the evolution of gas ceased, filtered hot, the filtrate acidified with hydrochloric acid, and the precipitate collected, washed and dried. It appeared then as a pale yellow powder, unmelted at 302°; yield, 25.9%.

Anal. Calc. for $C_{13}H_{10}O_3NAsS$: C, 46.57; H, 2.98; As, 22.4. Found: C, 47.35; H, 3.48; As, 20.8.

Mononitro-2-phenyl-benzothiazole-p-arsonic Acid.—A solution of 4.41 g. of 2-phenyl-benzothiazole-*p*-arsonic acid in 50 cc. of concentrated sulphuric acid was nitrated at 40–45° with a solution of 0.84 cc. of concentrated nitric acid (sp. gr., 1.42) in 10 cc. of concentrated sulphuric acid. The clear reddish brown solution was kept at 40–45° for 15 minutes, and then poured into 10 volumes of ice water. The precipitated nitro derivative, when removed, washed and dried formed a dull yellow powder; yield, 4.7 g., or 94%.

Anal. Calc. for $C_{13}H_9O_5N_2AsS$: C, 41.05; H, 2.37; As, 19.7. Found: C, 43.51; H, 2.62; As, 22.0.

These analytical figures indicated the presence of some un-nitrated initial material and on reduction this proved to be the case. The un-nitrated material remained insoluble in the acidified reduced solution and was removed.

A subsequent nitration, conducted at 50–60°, gave a slightly darker product, but one which separated no insoluble material in the acidified reduced solution.

In view of the reactivity of Position 6, it is our belief that the nitro group has probably entered there.

Monoamino-2-phenyl-benzothiazole-p-arsonic Acid was obtained by reducing the nitro acid by the method of Benda,[4] as modified by Jacobs, Heidelberger and Rolf,[5] using ferrous sulphate and alkali, which effects the reduction of the nitro group without changing pentavalent arsenic to trivalent. The dry product formed a grayish green powder; yield, 24.3%.

Anal. Calc. for $C_{13}H_{11}O_3N_2AsS$: C, 44.7; H, 3.15; As, 21.4. Found: C, 45.29; H, 3.39; As, 21.8.

It was very slightly soluble in water or alcohol with greenish fluorescence, insoluble in ether or toluene, and soluble in alkali to a brown solution which exhibited a greenish fluorescence when diluted. Its solution in hydrochloric acid was of a faint rose color. When this solution was made alkaline with ammonium hydroxide, magnesium chloride and ammonium chloride added and heat applied, a fine pale yellowish precipitate separated.

Diazotized and coupled with β-naphthol in alkaline solution, this amine yielded a cherry-red dye.

Through the courtesy of Dr. A. E. Sherndal of the H. A. Metz Laboratories, Inc., some preliminary pharmacological tests were made on this amino arsonic acid, which is soluble in normal sodium hydroxide and in this form was suitable for intravenous injection. These tests indicated that the compound has a somewhat greater, though slower, action upon *Tr. equiperdum* than the corresponding benzene derivative, and also a greater toxicity for the animals (white rats) used in the experiments. No evidence of any effect upon the nervous system was noted at any of the dosages used. If this preliminary observation is confirmed by further experiments it may prove of considerable interest, since the well known powerful action upon the nervous system has been one of the most objectionable features of many of the pentavalent arsenic compounds.

2-(p-Hydroxyphenyl)-benzothiazole Arsonic Acid.—A suspension of the difficultly soluble reddish orange diazonium chloride from 1.14 g. of the monamino-2-(*p*-hydroxyphenyl)-benzothiazole was added to a mixture of 0.95 g. of arsenic trioxide, 14 g. of potassium hydroxide and 1 g. of copper bronze, in 200 cc. of ice water. The temperature was kept at 3–5° during the mixing, after which it was allowed to rise gradually to that of the room, and finally was heated nearly to boiling until evolution of gas ceased. The dark red solution was filtered and the filtrate acidified with hydrochloric acid. The deep red flocculent precipitate formed a dark chocolate powder when dry; yield, 0.77 g. or 46.7%.

Anal. Calc. for $C_{13}H_{10}O_3NAsS$: C, 44.44; H, 2.85. Found: C, 46.95; H, 3.24.

Qualitative tests showed the presence of arsenic. It dissolved freely in sodium hydroxide and but very slightly in alcohol. It was manifestly

impure but the amount of material available was insufficient for further purification.

Summary.—1. The following arsonic acids of 2-phenyl-benzothiazole have been synthesized and studied: the *p*-arsonic acid, a nitro and an amino derivative thereof, the 6-arsonic acid, and a 2-(*p*-hydroxyphenyl)-benzothiazole arsonic acid.

2. Preliminary pharmacological experiments have been conducted with some of these and results of interest obtained.

* Contribution from the Chemical Laboratories of Columbia University, No. 493.

[1] Bart (*a*) *Ger. Pat.*, 250,264 (1910); *Friedlander*, **10**, 1254 (1913); (*b*) *Ger. Pat.*, 254,092 (1910); *Friedlaender*, **11**, 1030 (1915); (*c*) *Ger. Pat.*, 268,172 (1912); *Friedlaender*, **11**, 1032 (1915); (*d*) *Ann.*, **429**, 55 (1922).

[2] Fourneau, *Bull. soc. chim.* [4], **37**, 1 (1925).

[3] Hunter, (*a*) *Chem. News*, **128**, 103 (1924); (*b*) **129**, 77 (1924).

[4] Benda, *Ber.*, **44**, 3300 (1911).

[5] Jacobs, Heidelberger and Rolf, *J. Am. Chem. Soc.*, **40**, 1580 (1918).

THE ORIGIN OF TERRESTRIAL HELIUM AND ITS ASSOCIATION WITH OTHER GASES

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Read before the Academy April 27, 1925

The origin of gases in the earth's crust and atmosphere is of great interest and has been the subject of much speculation. While the present knowledge of the occurrence of gases in nature is by no means complete, helium is unique in offering avenues of approach that are lacking for other gases.

The chemical inertness of helium seems to preclude its forming compounds with other elements—at least under ordinary conditions—hence we exclude at once the possibility either of its genesis or of its removal by chemical action. This is likewise true for all of the other inert gases and would limit us to a theory of primordial origin. In the case of helium however, we have the possibility of radioactive origin which is so plausible as to appear to throw the burden of proof upon all other theories.

Terrestrial helium was first found by Sir Wm. Ramsay[1] in association with uranium, which a little later was discovered by Becquerel[2] to be radioactive. When it was later proved by Rutherford[3] that the α-rays emitted by uranium and other radioactive elements are high velocity helium nuclei, and that upon electrical neutralization they become ordinary atoms of

helium, the chain of evidence appeared complete and no question was raised as to the radioactive origin of helium, until several years later when vast stores of it began to be discovered in certain natural gases in the United States west of the Mississippi River.

In an extended report on helium-bearing natural gas in 1921, Rogers[4] reviewed the evidence for and against both primordial and radioactive origin. While he did not question the radioactive origin of helium actually occurring in uranium and thorium minerals, he found difficulty in accepting the same origin for the larger quantities found in natural gases and the vast total quantity in the atmosphere which Van Orstrand has calculated to be 58×10^{12} cubic meters. It was not so much that the total quantity to be accounted for appeared too great, but Rogers found difficulty in conceiving a mechanism by which so large a proportion of helium could be liberated from the original minerals or rocks as to escape into the atmosphere or be collected locally in gas bearing structures in the crust.

It may now be regarded as quite thoroughly established by the researches of Joly[5] and others that the radioactive elements are distributed through the known crust of the earth with surprising uniformity. Before considering the data quantitatively, it will be profitable to examine the conclusions that may be drawn from this general and even distribution of radioactive material. A like even distribution of helium generation is a direct consequence, which at once raises the question as to why its occurrence in quantity is not also evenly distributed, instead of being highly localized. Some authorities have attributed this localization of helium to local concentration of the parent radioactive elements. The alternatives would be localized conditions especially favorable to either: (1) its liberation from, (2) its migration to and collection in, or (3) its retention a local structure.

Many authorities including Rogers have been favorably inclined toward assuming localization of the radioactive source. While it is true that we do find deposits of uranium and thorium minerals of 1000 or even 10,000 times the general concentration in the crust, these deposits are quite as rare as those of helium, do not appear to be associated with them, and *in toto* constitute an insignificant part of the total radioactive content of the earth's crust.

Rogers discussed fully an alternative theory that a local deposit intermediate in radium content between an ore and the general crust will account for local concentration of helium without having to assume distant migration. This, while possible, is at present a pure assumption which appears unnecessary. Of the remaining alternative explanations of the localization of helium just proposed, all three may apply either singly or jointly and do not need to be discussed in detail. Of the three, it seems probable that the existence of local conditions suitable to the retention of

natural gas is the first essential. This does not explain the rarity of helium in natural gases. However, rarity must not be taken too literally. Rogers' report was based on the results of the analyses of gas from 325 sources. Up to the present the Bureau of Mines has extended the work to include about 1500 sources. Of these, only about 20% fail to give spectral evidence of helium. Half of them show more than 0.01%; and 1% of the gases contains more than 1% of helium by volume. This shows a very general distribution of helium in natural gases, but in widely differing concentrations. Therefore, the next most important factor after a retaining structure is the relative dilution of helium by other gases. This dilution may be partly effected in the original mineral, thus assisting in the primary removal of helium from it, or may have resulted subsequently through more or less accidental association with other gases, which may again assist migration by a "sweeping" action.

Regardless of whether we can explain the mechanism of removal of gas from the primary minerals or rocks, we have very direct and conclusive evidence of such leakage. The ratio of helium to lead in radioactive minerals furnishes the key. Both lead and helium are primary products of radioactive disintegration and are formed in a definite proportion to each other. If all the products of atomic disintegration were retained in place, one would find 8 atoms of helium for 1 of lead in uranium minerals and 6 atoms of helium for 1 of lead in thorium minerals. But actually never more than $^1/_2$ of this proportion of helium to lead is found and usually much less, as low as $^1/_{10}$, indicating that most of the helium either diffuses spontaneously from the minerals or is assisted by the sweeping action of more copious volumes of other gases.

The tectonic forces which assist migration of gases toward the earth's surface are little known and may be quite local or special in some cases. But if a general migration toward the surface is admitted, helium will eventually leak into the atmosphere or else be retained in an impervious structure. Such structures are not common, therefore escape by leakage into the atmosphere may represent the larger fraction, and through continuation over a period of years not exceeding 20 to 30 million could account for all the helium now in the atmosphere. We have no means of measuring the total leakage of helium or of any other gas into the atmosphere, but if it be so general as has just been indicated, it would appear necessary to assume continual diffusion out at the top,[6] or continuous removal into an upper helium rich atmosphere, thus accounting as transient the content of only 4 parts per million of helium in the lower atmosphere.

Of the three factors concerned, in the local accumulation of helium in the earth, (1) rate of generation, (2) accumulation period, and (3) rate of migration toward the surface, least is known of the latter. By consider-

ation of the two former we may at least see what limits must be set on migration to account for the results as far as they are known. Rogers' estimate of generation in the rocks of the crust at the rate of 0.5 cubic foot of helium per annum per cubic mile may be accepted. The more recent[7] data on the accumulation of helium in gas sands near the surface are of the same general character as the earlier ones with reference to concentration of helium, extent of sands, rock pressure and other factors, so that we may still regard the Petrolia field in Texas (see Rogers' Report) as a typical one. For an area of 10 square miles and a total helium content of 1 billion cubic feet, we have there an accumulation of 100 million cubic feet per square mile, which would represent the migration from a depth of one mile of the helium accumulated during 200 million years, or from 2 miles during 100 million years, etc. These dimensions of time and depth of migration do not appear at all excessive, without having to assume lateral or "inverted funnel" migration, nor any unusual radioactive processes.

Since the period estimated to attain the known accumulation in the lower atmosphere is so much shorter than that to stock a gas pool like Petrolia, it seems probable that a steady state has been reached in the lower atmosphere, unless local conditions of liberation and of migration play the major rôle.

The recent discoveries of helium in the United States indicate that its occurrence is not confined to limited geological horizons as earlier seemed probable. The occurrence now appears to be purely a matter of the probability of the retention of the gas. The favorable structure is a capping of shale with sufficient clay to afford a gas seal. Faulted or fissured structures are unfavorable. With reference to world distribution, it seems likely that when another area is so widely explored for gas and oil, as has been the case in the United States, helium in quantity will be discovered in other parts of the world.

The association of helium with other gases is interesting and of great potential importance. While helium is usually found in hydrocarbon gases, this is not universally true; we now have cases in two Western states of gases containing helium where the hydrocarbons are low or wholly lacking, nitrogen and carbon dioxide being the predominating gases. Quite early Cady and McFarland[8] observed that helium is always associated with a high though variable nitrogen content. The converse is not true. The question arises as to whether these associations are purely accidental. The fact that we find helium varying from spectral traces all the way to 2% (and rarely up to nearly 4%) by volume in natural gases, indicates accidental rather than casual association. The fact that we never find helium in higher concentrations may mean (1) that its migration requires a "sweeping" action of higher concentrations of other gases, as has been indicated in the foregoing, or (2) that the latter are so much more abundant

that helium always becomes diluted to some degree and that where we find the greater proportion of helium, the conditions for great dilution must have been absent. Doubtless both causes contribute to the result.

The almost universal association of helium with nitrogen appears significant. The varying ratio found suggests going back to the original minerals to eliminate subsequent contamination. Hillebrand[9] first proved the presence of nitrogen in gases collected from Glastonbury (Conn.) uranite. Ramsay, Collie and Travers[10] confirmed the presence of nitrogen in uranite obtained through Hillebrand from the same origin.[11] They found the gases to contain 11% of nitrogen, while the gases from clevite they found to contain no nitrogen. Dr. Foote[12] in recently referring to this association of nitrogen and uranium has considered two alternate modes of origin; mechanical occlusion and radioactive genesis of nitrogen from uranium. From the radioactive standpoint the writer agrees that a genetic origin from uranium is highly improbable. The complete breaking down of an atom as implied by Foote's hypothesis of 1 U $\longrightarrow$ 17 N ($238 = 17 \times 14$) is without radioactive parallel; while occlusion of external nitrogen in a dense mineral like uranite seems equally untenable.

A third theory is here proposed, namely, that the initial α-particle acts on some nitrogen compound within the mineral to decompose it, thus simultaneously generating both helium and nitrogen. A similar proposal of the writer was quoted by Rogers (loc. cit.) with reference to the assumption of the concentrated occurrence of uranium in shales below the helium bearing gas sands. We know shales to be rich in nitrogenous matter the decomposition of which would give both nitrogen and hydrocarbons. Rogers had some hesitancy in admitting this power of the α-particle, to cause chemical decomposition, but there can be no doubt of it. The decomposition of ammonia by α-particles has been repeatedly demonstrated,[13] that of nitrous oxide and of ammonia by Wourtzel,[14] and quite recently slight decomposition of cyanogen and hydrocyanic acid.[15] More complex compounds containing nitrogen would probably be all the more easily decomposed. The number of molecules decomposed by 1 α-particle (He^{++}) is also known to be large, a few hundred thousand for one, provided all the energy be expended on the substance being decomposed. Of course, this would not be the case in rocks or minerals, but nevertheless, no matter how unfavorable the conditions, some rate of decomposition would be maintained owing to the long life of uranium (or thorium), until all the decomposable material had been acted on. According to this theory we might have then any conceivable $He:N_2$ ratio. If nitrogen compounds were initially abundant in the mineral, many molecules would be decomposed by each α-particle, and the ratio of He would be very low. With time this initial quantity would become reduced, assuming that there were no means of continued restoration from without and a state would be

reached where only very rarely would an α-particle encounter a molecule containing nitrogen—until finally no nitrogen would remain.

The experimental evidence to test such a theory is very meagre. As already stated, Ramsay, Collie and Travers (loc. cit.) found 10% of N_2 and 90% of He in American (Connecticut) uranite, but found no nitrogen at all in clevite.

At the suggestion of the writer, gases have recently been collected from four additional specimens of unanium mineral;[16] two primary, uranite from Keystone, S. Dakota, and uraninite from Katanga, Belgian Congo; and two secondary—carnotite from Colorado and a mixture of curite and becquerelite from Katanga.

GASES COLLECTED FROM URANINITE AND SECONDARY URANIUM MINERALS BY SOLUTION IN DILUTE (1:6) SULFURIC ACID

	MINERAL	URANINITE		CURITE AND BECQUERELITE	CARNOTITE
	ORIGIN	KATANGA	KEYSTONE, S. D.	KATANGA	COLORADO, U. S.
% Total U		77.8	66.9	65.4	26.1
% UO_3		37.54	28.58	78.6	31.4
% UO_2		52.77	48.87	0	0
% PbO		7.02	16.42	(15.3)*	..
Total gas					
(% wt. of mineral)		0.264	0.08	0.164	0.02
(cc. per g. mineral)		9.65	2.1	0.95	0.14
(% vol. of gases)		92.0	82.9	<0.05	<0.05
He (% wt. of mineral)		0.159	0.031	0	0
N_2 (% wt. of mineral)		0.076	0.030	0.010	0.015
(% vol. of gases)		6.3	11.6	8.0	82.8**
CO_2 (% wt. of mineral)		0.024	0.009	0.107	0
(% vol. of gases)		1.3	2.2	57.6	0
O_2 (% wt. of mineral)		0.005	0.010	0.047	0.004
(% vol. of gases)		0.4	3.3	34.4	17.2**

* Calculated, assuming a 2:1 mixture of curite ($5UO_3.2PbO.4H_2O$)—Schoep, *Compt. rend.*, 173, 1186 (1921), and of becquerelite ($UO_3.2H_2O$)—Schoep, *Compt. rend.*, 174, 1240 (1922), as indicated by crystal examination and the UO_3 analysis.

** The N_2 and O_2 content of carnotite is so low and so close to that of air that its internal origin might be questioned.

The absence of helium in the gases from the two secondary minerals is striking. While a much lower helium content would be expected in the younger (secondary) minerals, it cannot be entirely absent; these negative spectral results mean that there must be less than 0.05% in the gases examined. Owing to the greater probability of the existence or entrance of foreign decomposable material into the more porous secondary minerals and the more ready escape of the gases generated, the resulting sweeping action would be expected to keep the helium at a very low value. This

further indicates that helium from the secondary mineral would be very highly contaminated.

It is quite notable that the results obtained on American uranite by Ramsay and his co-workers are fairly close to the present ones for African and South Dakota uraninites of different geological ages. A genetic relationship (as suggested by Foote) cannot be dismissed offhand, but analysis of the gas content of a large number of radioactive minerals should first be made.

The source of combined nitrogen in a mineral like uraninite is puzzling. Nitrogenous material of the character found in shale is hardly to be expected. Hillebrand (loc. cit.) considered quite possible the presence of uranium nitride.

The association of helium with hydrogen in many radioactive minerals was reported by Ramsay, Collie and Travers (loc. cit.). The decomposition of water into the elements by α-particles has been repeatedly demonstrated and thoroughly studied.[17] The presence of free *oxygen* in uranium minerals has not been previously reported, but is found in notable quantities in some of the present analyses. The usual absence of oxygen in uraninite can well be accounted for by the oxidation of material such as UO_2, which is abundant in uraninite. Hydrogen is absent in natural gases, although as pointed out recently by Lind and Bardwell[18] it must result from the action of α-rays on natural hydrocarbons or on water. It must subsequently become removed by reaction with some oxidizing material. Its almost complete absence in all natural gas (except of volcanic origin) is very remarkable.

In concluding, the writer desires to point out that the universal presence of α-radiation in the earth's crust imposes on a system containing substances capable of being acted on by this form of radiation, a condition of radiative equilibrium which is quite different from ordinary thermal equilibrium. The lower the temperature the greater the difference between radiative and thermal equilibrium may become. The low order of intensity of the radiation will be compensated by the length of the geological time intervals during which it is operative. At higher temperatures, owing to enhancement of the rate of ordinary chemical reactions, thermal equilibrium will prevail. Consideration of heterogeneous radiative equilibrium of this kind will be complicated but should not be overlooked in geo-chemical investigations.

[1] Wm. Ramsay, *Proc. Roy. Soc.*, **58**, 65 (1895).

[2] Henri Becquerel, *Compt. rend.*, **122**, 420, 501, et. sq. (1896).

[3] Rutherford and Royds, *Phil. Mag.*, **17**, 281 (1909).

[4] G. S. Rogers, *U. S. Geological Survey Professional Paper*, **121** (1921).

[5] J. Joly, *Radioactivity and Geology* (1909); *J. Chem. Soc.*, **125**, 897 (1924).

[6] Jeans (*Dynamic Theory of Gases*, 1921, pp. 345–6) regards this as excluded unless we assume an improbably high temperature for the upper atmosphere.

[7] Unpublished.

[8] *J. Am. Chem. Soc.*, **29**, 1521–6 (1907).

[9] W. F. Hillebrand, *U. S. G. S. Bull.* **78**, p. 43 (1891); *Am. J. Sci.*, **40**, 384 (1890).

[10] *J. Chem. Soc. Lond.*, **67**, 684 (1895).

[11] This origin is stated in private correspondence between Hillebrand and Ramsay, which was brought to my attention by Dr. C. E. Waters of the Bureau of Standards.

[12] P. D. Foote, *Nature*, **114**, 789 (1924).

[13] Cameron and Ramsay, *J. Chem. Soc.*, **93**, 966–92(1908). Usher, F. L., *Ibid.*, **97-I**, 389–405 (1910).

[14] E. Wourtzel, *Le Radium*, **11**, 332–47 (1919).

[15] Lind, Bardwell and Perry (unpublished results).

[16] The specimens were donated to the Committee of the National Research Council on "The Age of the Earth from Atomic Disintegration," the one from Keystone, S. D., by Mr. F. L. Hess of this Committee and the one from Katanga by Mr. Lechien of Brussels to Prof. Lane, Chairman. Mr. C. W. Davis of the U. S. Bureau of Mines at Reno, Nevada, has made complete analyses which will be published in *Am. J. Sci.* The gases were collected by Mr. Davis and analyzed by Mr. H. S. Kennedy of the Helium Laboratory of the Bureau of Mines in Washington.

[17] Duane and Scheuer, *Le Radium*, **10**, 33 (1913).

[18] *Science*, **60**, 364 (1924).

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